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Risk averse management on strategic multistage operational two-stage stochastic 0-1 optimization for the Rapid Transit Network Design (RTND) problem

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ABSTRACT

The Rapid Transit Network Design planning problem along a multi-period time horizon is treated by considering uncertainty in passenger demand, strategic costs and network disruption. The problem has strategic decisions about the timing to construct stations and edges, and operational decisions on the available network at the periods. The uncertainty in the strategic side is represented in a multistage scenario tree, while the uncertainty in the operational side is represented in two-stage scenario trees which are rooted with strategic nodes. The variability in the strategic cost along the time horizon as well as the variability in the lost passenger demand to the operational transit system current conditions could be very high. In order to avoid the negative impacts of low probability but high cost or high lost demand scenarios, some risk reduction measures should be considered. In this work the expected conditional stochastic dominance functional is modeled in two flavors. First, controlling the cost in the strategic scenarios in selected groups and clusters and second, controlling the lost passenger demand in the operational scenarios. Both flavors are time consistent.

KEYWORDS

Transportation, Rapid Transit Network Design, multistage multi-horizon scenario trees, 0-1 models, risk averse, matheuristic algorithms.

1 INTRODUCTION

Transportation systems are spatially distributed systems, which are vulnerable to different incidents that may occur. Despite the unpredictable nature of these incidents in terms of location, time and magnitude, effective mitigation methods should be designed from the very first strategic stage of design.

When designing a transport network, decisions are made according to an expected value for network state variables, such as infrastructure, vehicle, and traffic conditions, which are uncertain in a planning horizon of up to decades.

In order to find resilient network designs, different research approaches can be used, such as deterministic static, two-stage stochastic, multistage stochastic and robust optimization, among others. Robust optimization features solutions which are immune to data uncertainty [8, 23]. However, these solutions have been demonstrated to be too conservative and, then, expensive on a daily basis [11]. The key is that the recovery of the system in different operational scenarios in a given strategic scenario may not be as expensive as the introduction of traditional robustness concepts.

It is also well known that deterministic models do not provide high quality solutions if long planning horizons are considered, where variability of data is prominent. It should be pointed out that the optimization of the Risk Neutral (RN) model has the drawback of providing a solution that ignores the potential variability of the objective function value in the scenarios and, so, the occurrence of low-probability high-cost scenarios. Alternatively, risk averse measures could be considered.

This work aims at advancing the state-of-the-art of rapid transit network design by introducing a novel modeling approach for a stochastic recoverable robustness.

Review of the State-of-the-Art. In the context of rail Rapid Transit Network Design (RTND), a complete review is recently given in [27]. There is an extensive literature about deterministic RTND problems, where all parameters are assumed to be known with certainty [7, 10, 12, 13, 22, 25, 29, 30]. But, stochastic optimization is currently one of the most robust tools for decision making and broadly used in real-world applications in a wide range of problems from different areas (energy, finance, production, distribution, supply chain management, etc.). It is well known that an optimization (say, minimization) problem under uncertainty with a finite number of possible supporting scenarios has a Deterministic Equivalent Model (DEM). Traditionally, special attention has been given to optimizing the DEM by minimizing the objective function expected value in the scenarios, subject to the satisfaction of all the constraints, i.e., the so-called Risk Neutral (RN) approach. Note that large DEMs can be solved by using different types of decomposition approaches, e.g., see in [1]. There have been many attempts with two-stage problems in the field of RTND, which are approximations of real problems [11, 17, 26]. Other rail related problems have been also addressed with a two-stage RN approach [9, 15, 16, 28]. Recently, in a series of works [4–6], an alternative approach so-named Service Reliability is introduced for solving large-scale mixed 0-1 models with uncertain passenger demand in RTND.

Let us point out that the optimization of the RN model has the drawback of providing a solution that ignores the potential variability of the objective function value in the scenarios and, so, the occurrence of low-probability high-cost scenarios. Alternatively, risk averse measures could be considered. A computational comparison of some risk averse measures is presented in [2]. Several versions of the multistage mixed 0-1 time-inconsistent risk averse measure based on the Stochastic Dominance (SD) functional introduced in [18] have been presented in [19], and a

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time-consistent version of the multistage mixed 0-1 risk averse SD measure is introduced in [21].

For strategic problems (such as RTND), strategic decisions should not depend, even in part, on operational uncertainties in the previous periods. Long-term uncertainty, basically passenger demand and investment costs, should be represented in a multistage scenario tree, where short-term operational uncertainty, basically RTN elements' disruptions, should be represented by considering sub-trees rooted with the strategic nodes. The mixture of those trees may be named as (strategic) multistage (operational) multi-horizon tree. It is worthy to point out that its structure strongly impacts on the model design. Additionally, that type of model should also impact on the decomposition methodologies for problem solving in an affordable effort. Partially due the problem difficulty, there is not a wide literature on the subject. As we know [20, 24, 33] are the first works dealing with multistage multi-horizon trees. A specific application is presented in [33] for a gas transportation network, where the risk averse measure Average Value-at-Risk is considered. A multistage multi-horizon modeling is presented in [3] for an electricity transmission and generation network capacity expansion planning, where the risk averse measure Time Stochastic Dominance is considered. [14] is the first work as we know that addresses the RTND problem as a RN model in a multistage scenario tree by considering dependent stage-wise non-Markovian scenarios with a mixture of the sets of strategic and operational uncertain parameters.

In order to avoid the negative impacts of low probability but high cost or high lost demand scenarios, this work presents a strategic multistage operational multi-horizon 0-1 stochastic risk averse optimization model for the RTND problem. The expected conditional stochastic dominance functional is modeled in two flavors. First, controlling the cost in the strategic scenarios in selected groups and clusters and second, controlling the lost passenger demand in the operational scenarios. Both flavors are time consistent.

This short version of the paper is organized as follows. Section 2 presents the main elements of the scenario tree partitioned in the strategic multistage tree and the operational two-sage trees rooted in nodes in the strategic tree. Section 3 is devoted to the meta model where strategic and operational constraints are considered. Section 4 presents several time-consistent risk averse measures based on the stochastic dominance functional. And, Sections 5 and 6 sketch out the solution approach and computational experiments, respectively.

2 STRATEGIC MULTISTAGE AND OPERATIONAL TWO-STAGE SCENARIO TREES

For completeness let us consider the main elements of the problem inspired in [14, 20]. To represent the uncertainty a scenario analysis approach is used, where the scenario set can be visualized in a tree. Let E be the set of stages along the time horizon, $E = |E|$, T_e be the set of periods (usually, years, semesters) in stage e, for $e \in E$, T be the set of periods in the time horizon, such that $T = \cup_{e \in E} T_e$, $T = |T|$, and Ω be the finite set of representative strategic scenarios. A *scenario* $\omega \in \Omega$ is a particular realization of the uncertain strategic parameters along the time horizon, it is represented in the tree as a root-to-leaf path. A node of the strategic scenario tree represents an event, where it is assumed that the realization of the strategic uncertain parameters and strategic decision variables take place at the first

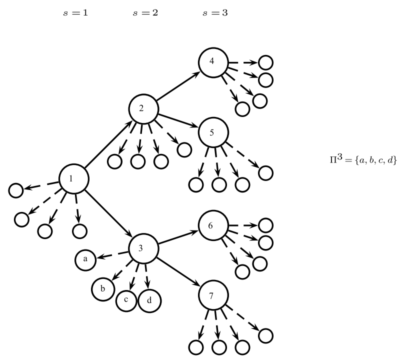

Figure 1: Strategic multistage scenario tree with operational two-stage scenario trees

period of the related stage. Notice that the group of scenarios that have the same realization of the uncertain parameters up to any given stage have the same value for the strategic decision variables up to that stage and, thus, the well-known *nonanticipativity* principle is satisfied. Let n and N denote a node and the set of lexicographically numbered nodes $\{1, \ldots, |N|\}$ in the tree, and N_e is the set of nodes that belong to stage e, such that $N = \cup_{e \in E} N_e$. Let also $\Omega^n \subseteq \Omega$ denote the group of scenarios with one-to-one correspondence with node n in the tree. Each node represents a point in time where a strategic decision can be made. Once a decision is made, some contingencies may occur, and information related to those contingencies is available at the beginning of the next stage.

The additional notation to represent the strategic multistage scenario tree is as follows:

- t_e, first period in the lexicographically ordered set T_e in stage e, for $e \in E$.
- e^n, stage to which node n belongs to, for $n \in N$.
- A^n, set of nodes composed of node n and its ancestors in the tree, for $n \in N$. Note: $A^1 = \{1\}$.
- $\tilde{A}^n$, set of nodes composed of node n and its ancestors whose related variables (i.e., representing strategic decisions) have nonzero elements in the constraints in node n, for $n \in N$. Note: $\tilde{A}^n \subseteq A^n$.
- S^n, set of successor nodes of node n in the tree, for $n \in N$.
- S_1^n, set of immediate successor nodes to node n, for $n \in N_e$, $e \in E$. Note: $S_1^n \subseteq S^n$.
- σ^n, immediate ancestor node to node n, thus, $\sigma^n \in A^n$, for $n \in N \setminus \{1\}$.
- w^ω, weight or probability assigned to scenario ω, for $\omega \in \Omega$, and $w^n = \sum_{\omega \in \Omega^n} w^\omega$, for $n \in N$.

Now, consider any node $n \in N$ also as a representative of any operational period of stage e^n. The operational uncertainty attached to node n is represented by a finite set of scenarios. They are so-called operational scenarios in a two-stage tree rooted with node n, and the realizations of the scenarios are, precisely, the nodes in the second stage. A 7-node scenario tree is depicted in Fig. 1.

In RTND problems, passenger demand may be considered as the most important uncertain parameter, since its uncertainty is the most independent one of the design of RTN; so, the strategic multistage scenario tree is generated around it. Also assume that the strategic (investment) cost is on some way correlated with the passenger demand. Therefore, passenger demand and investment cost are strategic uncertain parameters defined in strategic nodes while RTN disruptions and operational cost are operational uncertain parameters defined in operational nodes (the ones in the second stage of the two-stage tree rooted with strategic nodes). Thus, in order to have affordable dimensions in the scenario tree from a computational point of view, as an illustration consider $E = 4$ stages, with 5 periods, say years, each one. On the other hand, assume that the number of strategic immediate successor nodes of node n is $|S_1^n| = 3$ for $e^n = 1, 2, 3$ and, so, the cardinality of the scenario tree is $|N| = \sum_{e \in E} |N_e| = 1 + 3 + 9 + 27 = 40$ nodes. Some additional notation related to the RTN infrastructure is as follows:

- I, set of RTN infrastructure elements to be constructed.
- ℓ_i, latency, i.e., number of periods that are required between the period when the construction starts for the RTN infrastructure element i (e.g., an edge as a connection of two stations, a station in the network) and the period at which it becomes available for operation.
- I_i, set of RTN infrastructure elements whose construction cannot start until element i is available (i.e. its construction is over), for $i \in I$. Note: $I_i \subset I$.
- J, set of RTN operational elements.
- I^j, set of RTN infrastructure elements that should be available when operational element j is active, for $j \in J$. Note: $I^j \subset I$.

The notation for the other elements in strategic node n and its operational two-stage scenario tree is as follows, for $n \in N$:

- ι_i^n, strategic ancestor node related to RTN infrastructure element i, such that the period which it belongs to (i.e., t_e where $e \equiv e_{\iota_i^n}$) is the *latest* period *by* which element i can start its construction, so that it is available for use in the RTN at any period in set T_{e^n} for strategic node n, for $n \in N,\ i \in I$:

$$\iota_i^n = argmax_{q \in A^n}\{t_{e^q} \in T : t_{e^q} \leq t_{e^n} - \ell_i\}.$$

- Π^n, set of operational scenarios for the two-stage tree rooted with strategic node n. As an illustrative case, assume $|\Pi^n| = 8$ operational scenarios per each node n in the tree with $|N| = 40$ strategic nodes in the case that have been illustrated above. So, in total, there are 320 uncertain situations to be dealt with, being partitioned in 40 groups. It means that there are 320 RTN operational submodels within the strategic-operational one to be presented next. Many of those submodels will probably have the same or a similar topology.
- w^π, weight or probability of operational scenario π, for $\pi \in \Pi^n$, such that $\sum_{\pi \in \Pi^n} w^\pi = 1$.

3 STRATEGIC MULTISTAGE OPERATIONAL TWO-STAGE STOCHASTIC RISK NEUTRAL 0-1 MODEL

The Risk Neutral (RN) model that is introduced in this section requires the following notation for the variables:

- $(x^n)_i$, 0-1 *step variable* for RTN infrastructure element i in node n, for $i \in I$. Its value is 1 if the element starts its construction *by* period t_{e^n} and otherwise, 0, for $n \in N : t_{e^n} \leq T - \ell_i,\ i \in I$. It is a strategic variable. Let x^n be the $|I|$-dimensional vector of variables $\{(x^n)_i\ \forall i \in I\}$. Notice that $(x^n)_i - (x^{\sigma^n})_i = 1$ means that element i starts is construction *at* node n.
- $(y^\pi)_j$, 0-1 *impulse variable* for RTN operational element j in operational node π, for $\pi \in \Pi^n,\ n \in N,\ j \in J$. Its value is 1 if the element is active *at* operational scenario π in stage e^n to which strategic node n belongs to and otherwise, 0. It is an operational variable. Let y^π be the $|J|$-dimensional vector of variables $\{(y^\pi)_j\ \forall j \in J\}$.

Note: It is well-known that the modeling scheme where the step x-variables are considered is stronger than the model where they are replaced with impulse variables.

The parameters are as follows:

- $(a^n)_i$, objective function coefficient (i.e., investment cost) related to the RTN infrastructure element i if it starts its construction *at* node n, for $n \in N : t_{e^n} \leq T - \ell_i,\ i \in I$. It is assumed that the construction cost is made at the starting period t_{e^n}. Note: That assumption can be easily replaced with an ad-hoc policy.
- b^π, vector of the objective function coefficients (e.g., passenger demand lost, among others) of the operational variables in vector y^π, for $\pi \in \Pi^n,\ n \in N$.
- h_s^n, rhs of the set of constraints related to strategic node n, for $n \in N$.
- A_n^q, constraint matrix for the variables in vector x^q of ancestor node q in the strategic constraints related to node n, for $q \in \tilde{A}^n,\ n \in N$.
- h_o^π, rhs of the set of constraints related to operational scenario π, for $\pi \in \Pi^n,\ n \in N$.
- B^π, constraint matrix for the variables in vector y^π, for $\pi \in \Pi^n,\ n \in N$.
- k, interest rate by period.

The DEM RN 0-1 model can be expressed as follows:

$$z = \min \sum_{i \in I} \sum_{\substack{n \in N: \\ t_{e^n} \leq T - \ell_i}} \frac{1}{(1+k)^{t_{e^n}}} w^n (a^n)_i ((x^n)_i - (x^{\sigma^n})_i) + \sum_{n \in N} \frac{1}{(1+k)^{t_{e^n}}} w^n |T_{e^n}| \sum_{\pi \in \Pi^n} w^\pi b^\pi y^\pi, \tag{1}$$

subject to

$$\begin{array}{ll}
\sum_{q \in \tilde{A}^n} A_n^q x^q = h_s^n & \forall n \in N \\
(x^{\sigma^n})_i \leq (x^n)_i & \forall n \in N : t_{e^n} \leq T - \ell_i,\ i \in I \\
(x^n)_{i'} - (x^{\sigma^n})_{i'} \leq (x^{\iota_i^n})_i & \forall n \in N : t_{e^n} \leq T - \ell_i,\ i' \in I_i,\ i \in I \\
(y^\pi)_j \leq (x^{\iota_i^n})_i & \forall \pi \in \Pi^n,\ n \in N,\ i \in I^j,\ j \in J \\
B^\pi y^\pi = h_o^\pi & \forall \pi \in \Pi^n,\ n \in N \\
(x^n)_i \in \{0,1\} & \forall n \in N : t_{e^n} \leq T - \ell_i,\ i \in I \\
(y^\pi)_j = 0 & \forall \pi \in \Pi^n,\ n \in N,\ j \in J \\
y^\pi \in \{0,1\} & \forall \pi \in \Pi^n,\ n \in N,\ j \in J.
\end{array} \tag{2}$$

4 RISK AVERSE EXPECTED CONDITIONAL STOCHASTIC DOMINANCE FUNCTIONALS

There are some risk averse approaches that deal with risk management [31], see a computational comparison in [2]. Among them, the Stochastic Dominance (SD)-based measures reduce the risk of the negative impact of the solutions in non-wanted scenarios in a better way than the others under some circumstances. See in [18, 32] its theoretical foundations, among others.

In a rapid transit network, the variability in cost along the time horizon in the strategic scenarios (where the operational scenarios are considered) and the variability in lost passenger demand in the operational scenarios for the related strategic node in the stages could be very high. In order to avoid the negative impact of the solution in the scenarios, mainly those with low probability and high cost or high lost demand, some risk reduction measures should be considered.

Time-consistency

Roughly, a risk-averse measure is time-consistent if the solution to be obtained from the submodel supported by a subtree rooted with a node at a given stage in a multistage scenario tree is the same as the one for that node and successors in the model supported by the full multistage scenario tree.

The rationale behind a time-consistent risk measure is that the solution value to be obtained in any node n and its successors for the related submodel "'solved"' at stage e^n should have the same value as in the original model "'solved"' at stage $e = 1$. Obviously, the RN model given by (1) and (2) and the model given by (1), (2) and (5) supported by the operational two-stage trees rooted at the strategic nodes are time-consistent. Additionally, it is not difficult to prove that the model given (1),(2),(3) and (5) is also time-consistent; in another context, see [21].

Section 4.1 presents the expected conditional stochastic dominance (ECSD) version for controlling the objective function value (i.e., the overall strategic-operational cost) in the *scenario groups* for modeler-driven subset of stages. Section 4.2 presents a stochastic dominance (SD) risk averse functional for controlling the objective function value (i.e., the overall strategic-operational cost) in a modeler-driven set of *scenario clusters*. The conditions to be satisfied by the SD functional in order to have the time-consistency property are also given. And Section 4.3 presents the ECSD version for controlling the lost passenger demand in the operational scenarios.

4.1 Objective function excess risk reduction for strategic scenario groups

The risk averse measure ECSD for the Net Present Value (NPV) of the expected objective function value composed of the expected investment cost on stations and edges of the new network (for short, expected strategic cost) and the expected operational cost of the available infrastructure elements of the new network for each strategic node in the whole time horizon requires the following additional sets of modeler-driven scenario groups and profiles:

- E^{St}, subset of stages in set E, whose scenario groups with one-to-one correspondence with strategic nodes (including the related operational ones) are to be considered.
- P^n, set of profiles for scenario group Ω^n, for $n \in N_e,\ e \in E^{St}$.

For each profile $p \in P^n$, let the following modeler-driven parameters:

- ϕ^p, objective function (i.e., cost) threshold in the whole time horizon to consider for any scenario in group Ω^n (i.e., group with one-to-one correspondence with strategic node n), where the operational scenarios in set Π^n are taken into account.
- $\tilde{s}^p$, upper bound of the expected cost excess over threshold ϕ^p for any scenario ω in group Ω^n.
- $\bar{s}^p$, upper bound of the expected cost excess over threshold ϕ^p in group Ω^n as a whole.

The profile contents are inspired in the second-order stochastic dominance functional induced by integer-linear recourse for multistage stochastic problems, see its time-consistent version in [21].

The variable for pair (ω, p), where ω is a strategic scenario in group Ω^n and p is the index of profile in P^n is as follows:

- $s^{\omega,p}$, continuous variable that takes the expected cost excess over threshold ϕ^p in strategic scenario ω in group Ω^n, where the operational scenarios in set Π^n are taken into account.

The objective function (i.e., overall strategic-operational cost) risk reduction ECSD constraint system can be expressed as:

$$\begin{array}{l}\displaystyle\sum_{i\in I}\ \sum_{\substack{q\in A^{\omega}:\\ t_{e^q}\le T-\ell_i}} \frac{1}{(1+k)^{t_{e^q}}}(a^q)_i((x^q)_i-(x^{\sigma^q})_i)+\\ \displaystyle\sum_{q\in A^{\omega}} \frac{1}{(1+k)^{t_{e^q}}}|T_{e^q}|\sum_{\pi\in\Pi^q} w^{\pi}b^{\pi}y^{\pi} - s^{\omega,p} \le \phi^p \text{ and}\\ 0\le s^{\omega,p}\le \tilde{s}^p \quad \forall\omega\in\Omega^n,\ p\in P^n,\ n\in N_e,\ e\in E^{St}\\ \displaystyle\sum_{\omega\in\Omega^n}(w^{\omega}/w^n)s^{\omega,p}\le\bar{s}^p \quad \forall p\in P^n,\ n\in N_e,\ e\in E^{St}.\end{array} \tag{3}$$

Notice that the key element in constraint system (3) is that those scenario-cross constraints are related to scenarios that belong to the same group at any stage in set E^{St}.

4.2 Objective function excess risk reduction for strategic scenario clusters

A risk reduction functional for the objective function value in scenario clusters is presented in this section with a similar scheme as the one presented in the previous section for the scenario groups with one-to-one correspondence with a modeler-driven stage subset. So, it has similar notation for the risk reduction profiles. The difference between both functionals is that, now, the strategic scenarios to consider are clustered according to a modeler-driven criterion. So, let C denote the set of scenario clusters, and Ω_c is the set of strategic scenarios in the cluster indexed with c, for $c \in C$. There is a high flexibility on the structuring of the clusters, i.e., (a) a scenario could belong to more than one cluster, and (b) more than one cluster may have one-to-one correspondence with the same strategic node.

Let P_c denote the set of profiles for scenario cluster Ω_c, for $c \in C$. So, for each profile $p \in P_c$, let the following modeler-driven parameters:

- ϕ^p, Objective function (i.e., cost) threshold in the whole time horizon to consider for any scenario in cluster Ω_c, where scenario ω, for $\omega \in \Omega_c$ includes the strategic nodes in it ancestor path down to the root node in the strategic multistage tree, A^{ω}, so that the operational scenarios in set Π^q are taken into account, for $q \in A^{\omega}$.
- $\tilde{s}^p$, upper bound of the expected cost excess over threshold ϕ^p for any scenario ω in cluster Ω_c.
- $\bar{s}^p$, upper bound of the expected cost excess over threshold ϕ^p in cluster Ω_c as a whole.

The variable for pair (ω, p), where ω is a strategic scenario in cluster Ω_c and p is the index of profile in P_c is as follows:

- $s^{\omega,p}$, continuous variable that takes the expected cost excess over threshold ϕ^p in strategic scenario ω in cluster Ω^c, where the operational scenarios in set Π^q are taken into account.

The risk reduction stochastic dominance constraint system related to the objective function (i.e., overall strategic-operational cost) can be expressed as:

$$
\begin{aligned}
&\sum_{i\in I} \sum_{\substack{q\in A^{\omega}:\\ t_{e^q}\le T-\ell_i}} \frac{1}{(1+k)^{t_{e^q}}}(a^q)_i((x^q)_i-(x^{\sigma^q})_i)+\\
&\sum_{q\in A^{\omega}} \frac{1}{(1+k)^{t_{e^q}}}|T_{e^q}| \sum_{\pi\in\Pi^q} w^{\pi} b^{\pi} y^{\pi} - s^{\omega,p} \le \phi^p \text{ and}\\
&0 \le s^{\omega,p} \le \tilde{s}^p \quad \forall \omega\in\Omega_c,\ p\in P_c,\ c\in C\\
&\sum_{\omega\in\Omega_c} (w^{\omega}/w_c) s^{\omega,p} \le \bar{s}^p \quad \forall p\in P_c,\ c\in C,
\end{aligned}
\tag{4}
$$

where $w_c = \sum_{\omega\in\Omega_c} w^{\omega}$ for $c \in C$.

Notice that the key element in constraint system (4) is similar to the one in system (3), here, those scenario-cross constraints are related to scenarios that belong to the same cluster.

It is worth pointing out that the risk reduction functional given in system (4) is a time-consistent one, provided that the following conditions are satisfied:

(1) The scenarios do not overlap in the clusters, i.e., $\Omega_c \cap \Omega_{c'} = \emptyset$ for any pair $c, c' \in C : c \neq c'$.
(2) Each scenario cluster Ω_c for $c \in C$ is included in some scenario strategic group Ω^n, i.e., $\exists n \in N_e : e \in E$ such that $\Omega_c \subseteq \Omega^n$, $c \in C$.

It is also worth pointing out that the time-consistency of the functional does not prevent that any scenario group is partitioned in several scenario clusters.

4.3 Risk reduction for the lost passenger demand at selected strategic nodes

The risk averse measure ECSD is specialized in this section for risk reduction in the operational scenario set Π^n, $n \in N$. Here, the function to consider is (the operational one related to) the passenger demand lost to the current transport system in the operational scenarios in the strategic nodes of a subset os stages.

The operational-based ECSD requires the following additional sets and elements for modeler-driven strategic nodes:

E^{Op}, subset of stages in set E, whose passenger demand lost is to be reduced. Note: $E^{St} \cap E^{Op}$ could be an empty set.

W, passenger groups defined by origin/destination (o/d) pairs.

g^{π}_w, number of passengers in group w, for $w \in W$. Notice that its demand could be lost; it is a parameter that belongs to the objective function operational vector b^{π}.

f^{π}_w, a 0-1 variable, such that its value 1 means that passenger group w is lost to the current network in operational scenario π and otherwise, 0, for $w \in W$, $\pi \in \Pi^n$, $n \in N$. Note: Variable f^{π}_w belongs to operational variables vector y^{π}.

P^n, set of profiles that are associated with operational scenario set Π^n, for $n \in N^e$, $e \in E^{Op}$, instead of been associated with strategic scenario group Ω^n as it is presented in Section 4.1.

For each profile $p \in P^n$, the following parameters are required:

γ^p, passenger demand lost threshold to consider in any operational scenario π, for $\pi \in \Pi^n$.

$\tilde{s}^p$, upper bound of the demand lost excess over threshold γ^p in any operational node π, for $\pi \in \Pi^n$.

$\bar{s}^p$, upper bound of the expected demand lost excess over threshold γ^p in set Π^n.

The variable for pair (π, p), where π is an operational node and p is the index of a profile in strategic node n, for $p \in P^n$, $\pi \in \Pi^n$, $n \in N^e$, is as follows for stage e, for $e \in E^{Op}$:

$s^{\pi,p}$, continuous variable that takes the passenger demand lost over threshold γ^p in operational scenario π, for $\pi \in \Pi^n$.

The risk reduction ECSD constraint system related to the (operational) passenger demand lost can be expressed

$$
\begin{aligned}
&\frac{1}{(1+k)^{t_e}}|T_e| \sum_{w\in W} g^{\pi}_w f^{\pi}_w - s^{\pi,p} \le \gamma^p \text{ and}\\
&0 \le s^{\pi,p} \le \tilde{s}^p \quad \forall \pi\in\Pi^n,\ p\in P^n,\ n\in N^e,\ e\in E^{Op}\\
&\sum_{\pi\in\Pi^n} w^{\pi} s^{\pi,p} \le \bar{s}^p \quad \forall p\in P^n,\ n\in N^e,\ e\in E^{Op}.
\end{aligned}
\tag{5}
$$

So, the ECSD model that is proposed in this work can be expressed as the expected cost (1) to minimize, subject to the strategic node-based constraint system (2), the one for linking strategic and operational variables and the operational node-based constraint system, plus the cross strategic scenario group and operational set based constraint systems (3) and (5), respectively.

5 SOLUTION APPROACH

Given the problem's complexity and the huge model's dimensions (due to the RTND static model as well as the potentially high number of scenario nodes in the multistage setting), it is unrealistic to seek for an optimal solution, even by considering decomposition approaches for problem solving. So, a decomposition approach is required for obtaining a (hopefully, good) feasible solution where its optimality gap is guaranteed.

A version of the matheuristic FLAggA (that stands for Fix-and-Lazy Aggregated / de-aggregated Algorithm) [14] is presented in the full paper to deal with the risk averse measures represented in the constraint systems (3), (4) and (5).

6 COMPUTATIONAL EXPERIMENT

This section introduces the computational experiment, whose results are not detailed due to space limitations. It is based on the RTN so-called R1, see [29], which has also been used in [10, 11, 17, 25], among others. It features 9 nodes, 15 edges and 72 passenger groups. All previous efforts for problem solving have been devoted either to the deterministic or the RN version of the network.

A broad computational study is performed to compare the performance of FLAggA and the plain use of a state-of-the-art solver on one hand. And on the other one, a computational analysis is carried out by comparing the RN version of the model with the proposed risk averse measures.

Two different scenario trees are considered in the experiment, namely a proof-of-concept tree and a tree with more realistic dimensions. The first tree features 3 stages, 7 strategic nodes, 56 operational scenarios and 4 strategic scenarios. The second tree features 4 stages, 40 strategic nodes, 320 operational scenarios and 27 strategic scenarios.

The RN solution provides a high variability in many issues, as for example in the lost passenger demand. For the 40-node scenario tree case, for illustrative purposes, the differences between the strategic scenarios is shown in Figure 2. The upper

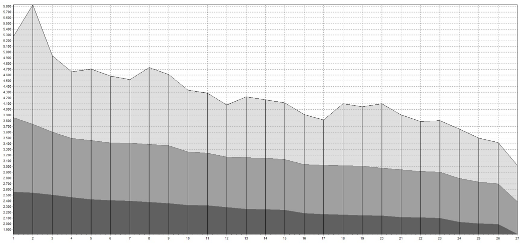

Figure 2: Scenario demand and lost demand

curve in the figure depicts the passenger demand for each of the strategic scenarios, while the middle and lower curves depict the expected lost of passenger demand for two latency strategies as provided by the incumbent solution obtained by the matheuristic algorithm FLAggA.

Table 1 shows some statistics for the demand and lost demand in the scenarios. The headings are related to the largest, smallest, average and its standard deviation. The purpose of the proposed risk averse functionals is to reduce this level of lost demand with the same allowed budget for infrastructure investment.

Table 1: Passenger demand statistics for the 27 strategic scenarios

Demand	*largest*	*smallest*	*aver*	*dev*
Scenario-based	5828.52	3025.38	4229.25	577.64
Lost for $\ell = 1$	3858.39	2391.86	3160.52	325.99
Lost for $\ell = 0$	2560.68	1823.13	3221.41	179.47

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