Incremental Based Top-k Similarity Search Framework for Interactive-Data-Analysis Sessions

Oded Elbaz, Tova Milo, and Amit Somech

Tel Aviv University, Israel

ABSTRACT

Interactive Data Analysis (IDA) is a core knowledge-discovery process, in which data scientists explore datasets by issuing a sequence of data analysis actions (e.g. filter, aggregation, visualization), referred to as a session. Since IDA is a challenging task, special recommendation systems were devised in previous work, aimed to assist users in choosing the next analysis action to perform at each point in the session. Such systems often record previous IDA sessions and utilize them to generate next-action recommendations. To do so, a compound, dedicated session-similarity measure is employed to find the top-k sessions most similar to the session of the current user. Clearly, the efficiency of the top-k similarity search is critical to retain interactive response times. However, optimizing this search is challenging due to the non-metric nature of the session similarity measure.

To address this problem we exploit a key property of IDA, which is that the user session progresses *incrementally*, with the top-k similarity search performed, by the recommender system, *at each step*. We devise efficient top-k algorithms that harness the incremental nature of the problem to speed up the similarity search, employing a novel, effective *filter-and-refine* method. Our experiments demonstrate the efficiency of our solution, obtaining a running-time speedup of over 180X compared to a sequential similarity search.

1 INTRODUCTION

Interactive Data Analysis (IDA) is an important procedure in any process of data-driven discovery. It is ubiquitously performed by data scientists and analysts who interact with their data "hands-on" by iteratively applying analysis actions (e.g. filtering, aggregations, visualizations) and manually examining the results. This is primarily done to understand the nature of the data and extract knowledge from it, yet is also fundamental for particular data scientific tasks such as data wrangling and cleaning, feature selection and engineering, and explaining decision models.

However, since IDA is long known as a complex and difficult task, extensive research has been devoted to the development of *recommendation systems* that assist users in choosing an appropriate next-action to perform at each point in an IDA session. Many of these IDA recommendation systems [2, 15, 16, 26, 43] rely on a *similarity comparison* between users' sequences of analysis actions (denoted *sessions*). They follow the assumption that if two session *prefixes* are similar, their *continuation* is likely to also be similar. Hence, they utilize a repository of prior analysis sessions (of the same or other users): Given an *ongoing* user's session, such systems first retrieve the top-k most similar session prefixes from the repository, then examine their continuation and use the gathered information to form a possible next-action recommendation [2, 15, 16, 43]. Since these recommender systems are *interactive*, the efficiency of the top-k session similarity search is critical. However, most previous work focuses on the *quality* and *applicability* of the produced IDA recommendations, rather than on their *scalability* and running time *performance*.

Our goal in this paper is thus to devise efficient, scalable algorithms for this particular top-k similarity search problem, a significant computational bottleneck in many IDA recommender systems. Specifically, we focus our attention on a dedicated similarity measure for analysis sessions [3], which we denote by *SW-SIM*. In a comprehensive user study [3], *SW-SIM* was compared, quality wise, against several alternative similarity measures, and was found to be the most suitable for the context of data analysis sessions. Consequently, it has been adopted by multiple IDA recommender systems and applications, e.g., [2, 4, 30, 41, 44]. *SW-SIM* is an extension of the well-known Smith-Waterman algorithm [35] for local sequence alignment. Intuitively, given a similarity metric for individual analysis-actions (e.g. filter, aggregation, visualization), *SW-SIM* compares two sessions s, s' by *aligning* them, i.e. matching similar action pairs using an *alignment matrix*. The measure allows for (yet penalizes) gaps in the alignment, and gives a higher weight to the more recent actions in the session, since they are expected to have higher relevance on the current user's intent.

Given a current user session and a sessions repository, a naive top-k search may be done by sequentially iterating over all sessions in the repository, computing the *SW-SIM* similarity score of the current session w.r.t. each of their prefixes, then selecting the top-k prefixes with the highest score. This simple algorithm, however, is prohibitively time consuming: **Our experiments show that even when employed on a medium size repository of 10K sessions, the naive sequential search takes more than 17 seconds to complete**. This is excessively high for a real-time response.

To optimize it, two key challenges must be addressed:

1. A single similarity comparison of two sessions is expensive to begin with. Computing *SW-SIM* requires to construct an alignment matrix, which results in a high computation time (quadratic in the mean session length).

2. Employing existing optimizations is highly non-trivial. Since *SW-SIM* is a non-metric similarity measure, existing optimizations for similarity search e.g. [8, 33, 37] are inadequate. Furthermore, because analysis sessions are compound sequences of complex analysis actions, they do not have a numeric vector representation, which is a requirement by other top-k optimization works [33].(See Section 2 for an elaborated discussion.)

A key observation underlying our work is that the user session progresses *incrementally* with the top-k similarity search performed by the IDA recommender system *at each step*. We exploit this property to address the two challenges mentioned above: Our first, rather direct optimization, speeds up the processing of a *single* similarity comparison between two sessions by utilizing

previous-step computations. Our second, more sophisticated optimization, tackles the *top-k search* by employing a novel filter-and-refine technique. It employs lower and upper bounds stemming from the incremental growth of the sessions accompanied by a dedicated index structure. Our experiments demonstrate that **using our optimizations, a speedup of more than 180X is obtained compared to a non-optimized sequential search**. Importantly, this is done while retaining a perfect accuracy of results, since our algorithms are exact.

We next explicitly state our assumptions and focus, then summarize our contributions and paper organization.

Paper Focus and Assumptions.

1. Although the particular IDA settings may vary, many IDA recommender systems rely on a top-k prefixes search. IDA settings may vary between systems, in terms of, e.g., the type of allowed analysis actions, structure of the data, user expertise, etc. However, the incremental, session-like nature of the process characterizes most IDA settings. Our paper is thus aimed at solving a computational bottleneck in a growing number of IDA-dedicated recommendation systems [2, 15, 16, 26, 43] that rely on finding the top-k most similar session prefixes in order to generate recommendations. Our solution is generic, suitable for a wide range of IDA platforms, regardless of their particular settings.

2. *SW-SIM* is currently the most-suitable measure for IDA sessions similarity. A multitude of definitions exist for measuring the similarity between two arbitrary sequences, such as *Dynamic Time Warping* for time series and *Smith-Waterman* for DNA sequences. In the context of data analysis sessions, *SW-SIM* is considered the most comprehensive and suitable (See [3] for a comparative study).

3. The focus of this paper is on performance and scalability, rather than on the quality of the produced recommendations. Different IDA recommender systems may use the selected top-k sessions in different ways in order to generate recommendations, possibly using additional information such as the data properties, user profiles, etc. But in all of them - response time optimization of the top-k search is clearly a critical issue. We therefore focus on performance and scalability rather than discussing the quality of the recommendations produced. For the latter, we refer the reader to works e.g. [2, 15, 16, 26, 43] in which the focus is on the quality of recommendations produced.

Technical Contributions & Paper Organization.

Section 3: We provide a simple, generic model for representing IDA sessions, and describe the alignment-based session similarity notion. Based on this model, we develop a formal definition for the incremental top-k similarity search problem.

Section 4: We describe an optimization for the incremental construction of a session alignment matrix and means to derive prefixes similarity scores from it.

Section 5: We present a novel threshold-based algorithm for the incremental top-k search, which utilizes effective similarity lower and upper bounds, also stemming from the incremental nature of the search problem.

Section 6: We demonstrate the efficiency of our solution via extensive experiments on artificial and real-life session repositories.

We begin by reviewing related work (Section 2), and finally present our conclusions and limitations in Section 7.

2 RELATED WORK

There is vast literature on sequences similarity, alignment, and top-k search. We survey works in numerous application domains, explaining why existing optimizations are inadequate for the context of IDA.

Bioinformatics-based optimizations for sequence alignment. Local alignment techniques are known to be extremely useful in the bioinformatics domain for tasks such as DNA and protein sequencing [22, 25, 39]. In this application domain, alignment is often performed between a query sequence and an extremely long *reference* string (e.g. a DNA genome), often from a fixed, small size alphabet (comprises, e.g., six nucleotides). Consequently, biology-driven optimizations such as [14, 18, 28, 40] exploit this particular property (small alphabet size) in order to index the large reference string. These solutions typically use a prefix/suffix tree or hash tables mapping small, common substrings to their location in the reference string. In contrast, in our context the "alphabet" (namely the analysis-actions space) is unbounded and, as explained in the next section, there is a predefined notion of distance between letters (analysis actions in our context). Such solutions, therefore, cannot be directly employed in our case.

Edit Distance and Dynamic Time Warping (DTW). Edit distance and DTW are popular alignment-based measures that are used in numerous application domains such as textual autocorrection, speech recognition and the comparison of time series. However, as noted in [3], these measures lack important properties for the context of IDA sessions. Classical edit-distance, which originated from the comparison of textual strings, assumes that letters are either *identical* or *mismatched*, which is inadequate for the context of IDA. This is because the alphabet, containing individual actions, is much larger and more complex. Also, the compound similarity of individual "letters" (actions) should be taken into consideration. DTW, on the other hand, does support the employment of a designated distance metric for the sequences' objects, yet is not compatible with the notion of *gaps* [32]. Also, both measures do not support the important time-discount feature [3], which allows giving a higher weight to the more recent actions in the session. Optimization works for these measures are typically based on specific properties of the measures or application domain [19, 38].

General solutions for similarity search in a non-metric space. *SW-SIM* is a non-metric similarity measure, thus many optimizations that rely on the *triangle inequality* property (e.g.,[8, 27]) cannot be used here. Other solutions for non-metric spaces, e.g. [6, 7, 19, 34], assume that the sequences' objects are numeric vectors (e.g. as in Minkowski distance, Cosine distance), as opposed to sequences of abstract objects (analysis actions in our case). But even ignoring this, employing any such solutions in our context requires indexing each session prefix, which may be prohibitively large (typically by an order of magnitude compared to the number of sessions). Existing solutions for optimized top-k search supporting arbitrary, non-metric similarity measure are Constant Shift Embedding (CSE) [31] and NM-Tree [34]. We examine these solutions in our experimental evaluation (Section 6.2) and show that our algorithm is consistently superior, obtaining a performance improvement of at least two orders of magnitude.

Incremental keyword query autocompletion. Incremental computation is used for optimization in a variety of database applications, e.g., interactive association-rules and sequence mining [29], but most notably for incremental text autocompletion [10, 17, 24]. the latter solutions efficiently perform a *top-k keyword*

similarity search at each key stroke of the user (using string edit distance). However, such solutions also exploit the rather small alphabet size and use a trie-index, a dedicated prefix tree for strings. As explained above, in our case the "alphabet" (i.e., the analysis-actions space) is complex and unbounded. Last, [21] and [20] suggest algorithms for incrementally computing the *string edit distance* of Ax and B (where x is an additional character) in $O(|A| + |B|)$ by using the distance matrix computed for A and B. We employ similar principles for our single-pair incremental alignment computation (Section 4), yet, importantly, augment them with novel techniques for efficient top-k similarity search.

Similarity search of scientific/business workflows. Similarity search has also been considered in the context of workflows [12, 23, 36] in two main contexts - searching for workflow *specifications*, and searching for workflow *traces*. They employ, however, a different notions of similarity focusing mainly on the structure/nesting of the workflow-specification components (different from the the unstructured, free-form nature of IDA sessions). Also, to our knowledge, these works do not address the *incremental* nature of the top-k similarity search problem, that is more typical for IDA sessions than for workflows.

3 PRELIMINARIES

We start by providing basic definitions for IDA, then define the incremental top-k similarity search problem.

Interactive Analysis Sessions. In the process of IDA, users investigate datasets via an interactive user interface which allows them to formulate and issue analysis actions (e.g., filter, grouping, aggregation, visualization, mining) and to examine their results. Formally, we assume an infinite domain[1] of analysis actions Q and model an analysis session as a sequence of actions $s = \langle q_1, q_2, \ldots, q_n \rangle | q_i \in Q$. We use $s[i]$ to denote the i'th action in s (q_i) and s_i to denote the session's *prefix* up to q_i, i.e. $s_i = \langle q_1, q_2, \ldots, q_i \rangle$. Therefore, $s = s_n$, and s_0 is the empty session. An ongoing user session, denoted u, is built incrementally. At time t the user issues an action q_t, then analyzes its results and decides whether to issue a next action q_{t+1} or to terminate the session. The session (prefix) u_t at time t thus consists of the actions $\langle q_1, \ldots, q_t \rangle$, where the following actions in the session, i.e., $q_{t+1}, ..q_n$ are not yet known.

In this work we consider the case of comparing the similarity of the user session u to other sessions (and parts thereof) in a given repository, denoted S, containing prior analysis sessions performed by the same or other users. To formally define sessions similarity, let us first consider the similarity of individual analysis actions, then generalize to sessions.

Individual Actions Similarity. Given a *distance metric* for individual analysis actions $\Delta : Q \times Q \rightarrow [0, 1]$ over the actions domain, the *action similarity function* $\sigma(q_1, q_2)$ is the complement function defined by $\sigma(q_1, q_2) = 1 - \Delta(q_1, q_2)$ for all q_1, q_2 in Q. Several distance/similarity measures for many kinds of analysis actions, e.g. for SQL and OLAP queries visualizations, and web-based analysis actions have been proposed in the literature (e.g. [3, 16, 26]) and our framework can employ any of them as long as the corresponding measure defines a metric space.

Session Similarity. As mentioned in Section 2, there are several ways to lift the similarity of individual elements into similarity of sequences [11]. To assess which measure is the most suitable for comparing analysis sessions, the authors of [3] formulated desiderata for an ideal session similarity measure, based on an in-depth user study:

- It should take the actions' *order of execution* into consideration, i.e. two sessions are similar if they contain a *similar* set of actions, performed in a *similar order*.
- "Gaps" (i.e subsequences of non-matching actions) should be allowed yet penalized.
- Long matching subsequences should be better rewarded than shorter matching subsequences.
- Recent actions in the sessions are more relevant than old ones.

They conclude that the only measure respecting all of the above mentioned desiderata is the *Smith-Waterman similarity measure* [35], a popular measure for local sequence alignment, and propose an extension suitable for the context of analysis actions, denoted *SW-SIM* in our work. We next describe the measure.

The similarity score of two sessions is defined recursively, on increasingly growing prefixes, with the base of the recursion being the empty prefix. At each point, the similarity of the pair of actions at the end of the prefixes is considered, and the best option, score-wise, is chosen. The two actions may either be matched, in which case an award proportional to their similarity is added to the accumulated score for their preceding prefixes, or, alternatively, one of the actions is skipped over. In this case a linear *gap penalty* $0 \leq \delta \leq 1$ is deducted from the accumulated score. To reflect the fact that the matching/skipping of older actions is less important than that of recent ones, rewards/penalties are multiplied by a decay factor $0 \leq \beta \leq 1$ with an exponent reflecting how distant the actions are from the sessions' end. More formally, the sessions similarity is defined using an *alignment matrix*.

Definition 3.1 (Alignment Matrix, Similarity Score). Given sessions s, s' of lengths n, m resp., their alignment matrix $A_{s,s'} \in \mathbb{R}^{(n+1)\times(m+1)}$ is recursively defined as follows. For $0 \leq i \leq n$, $0 \leq j \leq m$:

$$A_{s,s'}[i,j] = \begin{cases} 0, \text{ if } i = 0 \vee j = 0, \text{ else:} \\ max \begin{cases} A_{s,s'}[i-1, j-1] + \sigma(s[i], s'[j])\beta^{(n-i)+(m-j)} \\ A_{s,s'}[i, j-1] - \delta\beta^{(n-i)+(m-j)} \\ A_{s,s'}[i-1, j] - \delta\beta^{(n-i)+(m-j)} \\ 0 \end{cases} \end{cases}$$

The similarity score between s and s' is defined as:

$$Sim(s, s') := A_{s,s'}[n, m]$$

Note that even though the distance between *individual actions* forms a metric space, as noted in the introduction, *SW-SIM* does not induce a metric on IDA sessions.

		a	b	c	A	B
	0	0	0	0	0	0
A	0	0.24	0.19	0.13	0.66	0.58
B	0	0.19	0.53	0.47	0.58	1.47
A	0	0.30	0.47	0.53	1.28	1.38
B	0	0.23	0.66	0.58	1.19	2.28

(a) Session ϕ

		A	B	a	b	c
	0	0	0	0	0	0
A	0	0.48	0.43	0.37	0.30	0.23
B	0	0.43	1.07	1.00	0.93	0.85
A	0	0.59	1.00	1.43	1.35	1.26
B	0	0.52	1.32	1.35	1.88	1.78

(b) Session ψ

Figure 1: Alignment Matrices for u_4 = "*ABAB*"

[1]The domain is practically infinite since some action types, e.g. "filter", may have an unbounded number of possible parameter assignments.

The optimal values for the decay factor β and the gap penalty δ are typically chosen by the system administrator using an extrinsic evaluation, as explained in Section 6.1.

The following example illustrates the definition.

Example 3.2. For simplicity, assume that the action space is represented by the English letters, both uppercase and lowercase, and assume the following action similarity for any two distinct uppercase letters X,Y and their corresponding lowercase versions x,y.

$$\sigma(\cdot,\cdot) := \begin{cases} \sigma(X,X) = 1 \\ \sigma(x,x) = 1 \\ \sigma(x,X) = \sigma(X,x) = 0.5 \\ \sigma(x,y) = \sigma(X,Y) = 0.1 \\ \sigma(X,y) = \sigma(y,X) = 0 \end{cases}$$

Identical letters are maximally similar, and two instances of the same letter, but in different case (upper/lower), are 0.5 similar. Different letters but of the same case are 0.1 similar. Let u_4 ="ABAB" be the current user session, and consider two repository sessions ϕ ="abcAB" and ψ ="ABabc". Intuitively, according to the desiderata above, we expect ϕ to be more similar to u_4 than ψ, as their most recent suffix ("AB") is identical. This is reflected also in the alignment matrices of ϕ and ψ, depicted in Figure 1a and Figure 1b (resp.), when setting the gap penalty $\delta = 0.1$ and decay factor $\beta = 0.9$. The bottom-right cell in each matrix reflects the alignment score of the two sessions. The highlighted cells describes the alignment "trace", namely the cells chosen (among the three options in alignment formula) when advancing to the next step in the final score computation. Specifically, when the highlighted trace moves vertically/horizontally we have a gap, and when it moves diagonally the corresponding actions are matched. As we can see, the similarity score $Sim(u_4, \phi)$ is higher than $Sim(u_4, \psi)$.

Our definition of *SW-SIM* follows that of [3, 35] with a minor adaptation to our context. First, we define the similarity score as the *bottom-right* cell of the alignment matrix, as opposed to the *maximal* cell value in [3, 35]. This is because we focus on measuring the similarity of the current session to all *session prefixes* in the repository[2]. Second, we apply a decay factor to both actions matches and gaps, as opposed to only matches in [3], since their significance decreases as the session advances. Also, note that [3] suggests dynamically setting the decay factor and gap penalty, which are both constants in our case. As we shall see, the use of constant parameter values facilitates effective optimization via computation factorization.

Problem Definition. To assist the user in choosing an appropriate next-action, IDA recommender systems search the repository S to identify session prefixes that are most similar to u_t - the current, ongoing user session at time t. The continuation of the retrieved sessions is then processed by the recommender systems and used to derive a next-action recommendation for u_t.[3]

Let $\mathcal{P}refix(S)$ be the set consisting of all session prefixes in repository S, i.e. $\mathcal{P}refix(S) = \{s_i \mid s \in S, 1 \le i \le |s|\}$. Since the user session $u = \langle q_1, q_2, ...\rangle$ is built incrementally, at each step $t = 1, \ldots, |u|$ we are given u_t and wish to identify its top-k most similar session prefixes in the repository.

[2]This is done by IDA recommendation systems in order to process the continuation of matching session-prefixes to next-action recommendations.

[3]Other information is often used as well, e.g. the *interestingness* or *frequency* of the actions to be recommended, the user profile, etc.

u \ ϕ		a	ab	abc	abcA	abcAB
ABAB	0	0.35	0.90	0.71	1.32	2.28
ABABc	0	0.22	0.71	1.23	1.09	1.95

(a) u, ϕ

u \ ψ		A	AB	ABa	ABab	ABabc
ABAB	0	0.80	1.81	1.67	2.09	1.78
ABABc	0	0.62	1.53	1.47	1.78	2.19

(b) u, ψ

Figure 2: Similarity vectors for u_4 and u_5

Definition 3.3 (Incremental Top-k Similarity Search). Given a user session u_t at time t and a sessions repository S, the set $top_k(u_t, S) \subseteq \mathcal{P}refix(S)$ consists of k session prefixes s.t. $\forall s \in top_k(u_t, S), \forall s' \in \mathcal{P}refix(S) \backslash top_k(u_t, S) : Sim(u_t, s) \ge Sim(u_t, s')$. The incremental search problem is to compute, at each $t = 1, \ldots, |u|$, the set $top_k(u_t, S)$.

For simplicity, we assume that in the course of a user session u, the repository S is unchanged. In Section 5.5 we discuss the minor changes required in our framework to support the case of a dynamic repository where sessions are incremented or added.

Last, note that while session prefixes may be long and contain "historical" actions of less importance, *SW-SIM* (as explained above) favors *recent*, later actions over old ones. Also, it is easy to show that in *SW-SIM*, the similarity of a prefix $s_i = \langle q_1, q_2, ...q_i\rangle$ is always higher (or equal) than the similarity of any shorter *sub-session* of s ending in q_i.

4 INCREMENTAL SIMILARITY COMPUTATION

We first optimize the similarity calculations between the current user session u and all prefixes of a *single* repository session s. Rather than computing the alignment matrix from scratch for each prefix of s, we show that: (1) it is sufficient to compute one alignment matrix between two sessions, then use it to simultaneously derive *all prefixes similarity scores*, and (2) the alignment matrix $A_{u_t,s}$ at time t can be efficiently constructed by reusing the previous matrix $A_{u_{t-1},s}$ computed at time $t-1$.

Given a current user session u_t and a repository session s of size $|s|$, we define a *similarity vector*, $\vec{Sim}(u_t, s)$ containing the similarity score of u_t and each prefix of s, i.e.,

$$\vec{Sim}(u_t, s) = [Sim(u_t, s_0), Sim(u_t, s_1), \ldots, Sim(u_t, s_{|s|})]$$

We use $\vec{Sim}(u_t, s)[j]$, where $j = 1 \ldots |S|$, to denote the j element in the vector. The similarity vector $\vec{Sim}(u_t, s)$ can be derived from the alignment matrix $A_{u_t,s}$ using the following observation:

Observation 4.1. *Given an alignment matrix $A_{u_t,s}$*

$$\forall 0 \le j \le |s| : \vec{Sim}(u_t, s)[j] = \frac{A_{u_t,s}[t, j]}{\beta^{|s|-j}}$$

Namely, the similarity score of u_t and prefix s_j is derived from their corresponding element in the alignment matrix, $A_{u_t,s}[t, j]$, by readjusting the decay factor β. The proof is derived from the more general observation that for arbitrary prefixes s_i and s'_j of session s and s':

$$A_{s_i,s'_j}[i, j] = \frac{A_{s,s'}[i, j]}{\beta^{(|s|-i)+(|s'|-j)}} \quad (1)$$

Example 4.2. To continue with our running example, the similarity vectors $\vec{Sim}(u_4, \phi)$ and $\vec{Sim}(u_4, \psi)$ are given in the first row of the tables in Figure 2a, and Figure 2b, resp. According to Observation 4.1, an element in the similarity vector, e.g. $\vec{Sim}(u_4, \phi)[3]$, may be computed directly from the corresponding cell in the

alignment matrix of u_4 and ϕ: $\vec{Sim}(u_4, \phi)[3] = \frac{A_{u_4,\phi}[4,3]}{\beta^2} = \frac{0.58}{0.81} = 0.71$.

Further exploiting the incremental nature of the problem, we next show how to efficiently construct a similarity vector $\vec{Sim}(u_t, s)$ at time t, from that computed at time $t-1$, thereby avoiding explicitly building the entire alignment matrix. To do so, we generalize the dynamic programming construction of the alignment matrix (Definition 3.1). Vector entries at time t are computed by reusing entries in the previous similarity vector $\vec{Sim}(u_{t-1}, s)$, computed at time $t-1$ (see the colored parts of the formula in Proposition 4.3).

Proposition 4.3. *For every repository session s, user session u and time $t > 1$,*

$$\vec{Sim}(u_t, s)[j] = \begin{cases} 0, \text{ if } j = 0, \text{ else:} \\ max \begin{cases} \vec{Sim}(u_{t-1}, s)[j-1]\ \beta^2 + \sigma(u_t[t], s[j]) \\ \vec{Sim}(u_t, s)[j-1]\beta - \delta \\ \vec{Sim}(u_{t-1}, s)[j]\ \beta - \delta \\ 0 \end{cases} \end{cases}$$

The proof is obtained by employing Equation 1 on each case of the conditional definition.

Example 4.4. Continuing with our example, assume that the user now issues a new action "c", i.e. at $t = 5$, $u_5 =$ "*ABABc*". The new similarity vectors $\vec{Sim}(u_5, \phi)$ and $\vec{Sim}(u_5, \psi)$, are depicted in the bottom row of Figures 2a and 2b, resp. As before, the similarity scores $Sim(u_5, \phi)$ and $Sim(u_5, \psi)$, appear in the right-most cell of the vectors. Using Proposition 4.3 we can derive each value in the new vectors from the previous corresponding similarity vectors at time $t = 4$. For example, the fourth element in $\vec{Sim}(u_5, \psi)$ is given by:

$$\vec{Sim}(u_5, \psi)[4] = max \begin{cases} \vec{Sim}(u_4, \psi)[3]\beta^2 + \sigma(\text{“c”}, \text{“b”}) \\ \vec{Sim}(u_5, \psi)[3]\beta - \delta \\ \vec{Sim}(u_4, \psi)[4]\beta - \delta \end{cases}$$

$$= max(1.45, 1.22, 1.78) = 1.78$$

Let us analyze the reduction in time complexity resulting from these two simple optimizations. Let $|\widehat{s}|$ be the average session size and λ the complexity of computing similarity for individual actions. The expected time complexity of computing the similarity vector $\vec{Sim}(u_t, s)$ by construcitng all ($|\widehat{s}|$) allignemnt matrices is $O(|\widehat{s}|^3\lambda)$. Employing Observation 4.1 allows us to compute the same vector by constructing only one alignment matrix, in $O(|\widehat{s}|^2\lambda)$. Further employing Proposition 4.3 reduces the expected time to $O(|\widehat{s}|\lambda)$ since only $|s|$ action similarity calculations are required, between $u_t[t]$ (the new action in u_t) and all actions of s.

5 INCREMENTAL TOP-K ALGORITHMS

We are now ready to address the incremental top-k problem - given an ongoing user session u, retrieve the set $top_k(u_t, S)$ of similar prefixes at each time $t = 1, \ldots, |u|$. We first present a simple algorithm, denoted I-TopK, that iterates over the repository S and computes the similarity vector for each session. We then show how a much faster variant, denoted T-TopK, is obtained by employing a novel, incremental-based *filter-and-refine* approach.

Algorithm 1 T-TopK(u_t, S, k)

Input: u_t - current user session,
S - session repository,
k - size of the top-k set.
Output: $top_k(u_t, S)$ - a set of top-k most similar session prefixes to u_t.

```
1:  top ← MaxHeap(k)
2:  if t == 1 then
3:      Use I-TopK to obtain top_k(u_t, S)
4:  else
5:      Compute inf^t, the lower-bound similarity threshold
6:      C ← {s | s ∈ S ∧ sup^t(s) ≥ inf^t}
7:      for session s ∈ C do
8:          m ← latest time t' < t that Sim(u_t', s) was computed.
9:          for (i = m + 1; i ≤ t; i++) do
10:             Compute Sim(u_i, s) using Proposition 4.3
11:         for (j = 1; j ≤ |s|; j++) do
12:             top.push(Sim(u_t, s)[j], s_j)
13:         if |top| == k then
14:             C ← C \ {s | sup^t(s) < minScore(top)}
15: return top
```

5.1 Iterative Top-k Algorithm (I-TopK)

Algorithm I-TopK takes the following as input: the current session u_t, the sessions repository S, and the desired size k of the top-k set. It iterates over the repository, computing the similarity vector $\vec{Sim}(u_t, s)$ for each $s \in S$ by employing Proposition 4.3, i.e. by using the similarity vector $\vec{Sim}(u_{t-1}, s)$ calculated at the previous iteration ($t-1$). The top-k similarity scores (along with their corresponding prefixes) are maintained in a max-heap of size k, which will contain the exact set $top_k(u_t, S)$ of the top-k most similar prefixes to u_t at the end the loop.

The time complexity of I-TopK is $O(|S||\widehat{s}|\lambda)$, since it iterates over a session repository of size $|S|$ and computes a similarity vector in $O(|\widehat{s}|\lambda)$. The max-heap is maintained in a negligible cost of $O(log\ k)$, as we assume that $k << |S|$. As for space complexity, I-TopK requires storing the previous similarity vectors computed at $t-1$, therefore it requires $O(|S||\widehat{s}|)$ space, where $|\widehat{s}|$ is the average session size.

5.2 Threshold-Based Algorithm (T-TopK)

The T-TopK algorithm follows a *filter-and-refine* paradigm, s.t. in the *filter* step, repository sessions are pruned according to an *overestimation* of their similarity scores. Then, during the *refinement* step, the similarity vectors are computed only for candidate sessions passing the filter step. Importantly, T-TopK is guaranteed to be exact since the *filter* step always *overestimates* the true similarity scores and *underestimates* the filter thresholds. The novelty and efficiency of the algorithm stem from exploiting the incremental nature of the search problem.

We first describe the outline of the T-TopK algorithm, then in Sections 5.3 we present the techniques used for its *filter* step, i.e., the efficient calculation of the similarity lower and upper bounds.

T-TopK Algorithm Outline. Algorithm 1 depicts the outline of the T-TopK algorithm. For a given user session u_t, a sessions repository S and a number k, the prefixes set $top_k(u_t, S)$ is retrieved in the following manner. First, if u_t contains only one

action, the I-TopK algorithm will be used, since we have no previous prefix to rely on. Otherwise we employ a *filter-and-refine* process as follows.

Filter Step. We first find candidate sessions that are likely to contain prefixes in the top-k set $top_k(u_t, S)$:

(1) Form a global similarity threshold. We first compute an initial lower bound threshold, denoted inf^t, for the similarity score of a prefix to be a member of $top_k(u_t, S)$. The threshold inf^t *underestimates* the true minimal similarity score (Line 5).

(2) Compute a similarity upper-bound for each session. We then form a similarity upper-bound for each session s, denoted $sup^t(s)$, that overestimates the maximal similarity of a prefix in s to the current user session u_t.

The algorithm then filters the repository S by retrieving all sessions having similarity upper-bounds greater than inf^t (Line 6). Importantly, since the lower bound is *underestimated* and the upper bounds are *overestimated* - no false negatives can occur. In Section 5.3 we describe how the similarity lower and upper bounds are defined and efficiently calculated.

Refinement Step. We employ a *refinement* step over the candidate sessions and calculate the exact similarity scores using their *similarity-vectors*. Recall that to efficiently construct a similarity vector (Proposition 4.3) at time t, we need to have the vector of time $t-1$. However, we may not have calculated a similarity vector at $t-1$ if a session was pruned in the filter step. Thus, for each such candidate session we reconstruct, if necessary, its previous vectors from time $< t$ (We discuss below how this computation can be performed in user idle-times, allowing T-TopK a further speed-up), then compute the current similarity vector $\vec{Sim}(u_t, s)$ (Lines 9- 10).

Last, we iteratively push each element in the similarity vector into a max-heap *top* of size k (Lines 11- 12), and further prune candidate sessions if their upper bound is lower than the minimum score in *top* (Lines 13-14). Finally, the max-heap *top* holds the set $top_k(u_t, S)$, containing the top-k most similar prefixes to u_t.

Offline computation in user idle-times. User idle-times occur between two consecutive actions - while the user examines the results of her current action, before executing the next one (which typically takes several seconds). Our algorithm utilizes such idle-times, to compute similarity vectors skipped at previous iterations *offline*, so that they are already available when needed in the top-k search.

Correctness of the T-TopK algorithm. We next sketch the correctness proof, showing that the T-TopK algorithm is correct, i.e., always retrieves the *exact* set of top-k most similar session prefixes:

Let *top* denote the output of the algorithm. We need to show that $top_k(u_t, S) = top$. We will consider $t > 1$ (the case of t=1 is trivial). First, we show that for an arbitrary prefix s_j of a repository session s, $s_j \in top_k(u_t, S) \rightarrow s_j \in top$. If $s \in top_k(u_t, S)$ then $Sim(u_t, s) \geq inf^t$ (as inf^t is the similarity lower bound). Hence, for the upper bound $sup^t(s)$ of session s we can easily see that $sup^t(s) \geq inf^t$, thus $s \in C$, i.e. s is retrieved and processed as a candidate session. The proof for the case that $s_j \in top_k(u_t, S) \leftarrow s_j \in top$ follows similar lines.

To complete the picture we still need to explain how (1) the similarity lower bound inf^t, and (2) the upper bounds $sup^t(s)$ are computed and compared.

5.3 Incremental-Based Similarity Bounds

We first explain how the similarity lower and upper bounds are defined, then describe how the candidate sessions are efficiently retrieved.

5.3.1 Similarity Threshold (Lower Bound). The threshold inf^t forms a lower-bound (underestimated) for the similarity score of a prefix to be a member of $top_k(u_t, S)$. Intuitively, we use the sessions in the (already computed) top-k set of time $t-1$ as a potential representative for the top-k set of time t, and define our threshold w.r.t. them. Let $S^{t-1}_{top} \subseteq S$ denote the set of sessions with prefixes in $top_k(u_{t-1}, S)$. The similarity threshold is defined by:

$$inf^t = minScore(top_k(u_t, S^{t-1}_{top}))$$

As the size of S^{t-1}_{top} is at most k, inf^t is computed efficiently.

The proof that inf^t always underestimates the exact lower-bound can be immediately obtained from the simple observation that:

$$\forall S' \subseteq S, minScore(top_k(u_t, S')) \leq minScore(top_k(u_t, S'))$$

In particular, the observation holds for $S' = S^{t-1}_{top}$.

Nevertheless, we show that inf^t cannot be too far from the *exact* threshold at t-1, using the following observation:

Observation 5.1. $minScore(top_k(u_{t-1}, S))\beta - \delta \leq inf^t$

This stems from the fact that the lowest possible inf^t is obtained when the last query in u_t is not aligned in *any* of the sessions in S^{t-1}_{top} (hence a gap penalty is absorbed).

Example 5.2. Assume that $k = 1$ (i.e. we are interested in the top-1 similar prefixes), and consider the sessions u, ϕ, ψ from our running example. Considering the similarity vectors at $t = 4$ (detailed in Example 4.2), the most similar prefix is ϕ_5 and thus $top_1(u_4, \{\phi, \psi\}) = \{\phi_5\}$. Consequently, its corresponding session is in the set $S^4_{top} = \{\phi\}$. For computing the threshold $inf^5 = minScore(top_1(u_5, S^4_{top}))$, we construct the similarity vector $\vec{Sim}(u_5, \phi)$ (bottom row in Figure 2a), and take the maximal score among its elements, thus $inf^5 = 1.95$.

5.3.2 Upper-Bounding Similarity Scores.

Let $\widehat{sup}^t(s) = max_{1 \leq j \leq |s|} Sim(u_t, s_j)$ denote the *tight*, exact upper bound for the similarity score of all prefixes in s. Recall that for sessions in S^{t-1}_{top}, we already computed their similarity vectors when computed the lower bound inf^t, therefore their tight bound is already at hand. Nevertheless, for the rest of the sessions, i.e. $s \in S \setminus S^{t-1}_{top}$, we show that the incremental nature of the computation can be used to form $sup^t(s)$, an effective over-approximation to $\widehat{sup}^t(s)$. An important observation is that we are only interested in upper-bounding the scores of sessions containing at least one prefix with a similarity scores higher than inf^t (otherwise it can be safely pruned). We therefore define a *proper* similarity upper bound as follows:

Definition 5.3 (Proper upper bound). $sup^t(s)$ is a *proper upper bound* w.r.t. u_t and S, if for every session s having some prefix s_j where $Sim(u_t, s_j) > inf^t$, we have that $sup^t(s) \geq \widehat{sup}^t(s)$.

A second important observation, is that unless the last action in u_t, denoted q_t, is aligned to an action in $s \in S \setminus S^{t-1}_{top}$, s can be safely pruned.

Observation 5.4. *For a session $s \in S \setminus S^{t-1}_{top}$, if when computing $Sim(u_t, s)$ the action q_t is not matched with any of s's actions, then for every prefix s_j of s, $Sim(u_t, s_j) \leq inf^t$.*

This is because if q_t is not matched with an action in s_j it absorbs a gap penalty, i.e., $Sim(u_t, s_j) = Sim(u_{t-1}, s_j)\beta - \delta$ (Proposition 4.3). On the other hand, since $s_j \notin top_k(u_{t-1}, S)$ we know that $Sim(u_{t-1}, s_j) \leq minScore(top_k(u_{t-1}, S))$. Then, using Observation 5.1 we can see that $Sim(u_t, s_j)$ can never exceed inf^t.

Consequently, in the analysis below we ignore sessions where q_t is not matched with any of s's actions, and for brevity, unless stated otherwise, whenever we refer to a session s we mean one in $S \setminus S^{t-1}_{top}$ where q_t has a match.

We next provide two ways to overestimate $\widehat{sup}^t(s)$. In our efficient implementation of the filter procedure for candidate session retrieval (to be detailed in the next subsection), we use the first bound to quickly prune irrelevant sessions, then use the second one for further pruning the candidates set.

First bound (B_1). The following observation shows that we can bound $\widehat{sup}^t(s)$ from above using the top-k set of the previous iteration and the maximal similarity of the session's individual actions to the new action q_t.

Observation 5.5.

$$\widehat{sup}^t(s) \leq minScore(top_k(u_{t-1}, S))\beta^2 + max_{q \in s}\{\sigma(q_t, q)\}$$

Intuitively, this is because for every prefix not in $top_k(u_{t-1}, S)$, the similarity to u_{t-1} is smaller than $minScore(top_k(u_{t-1}, S))$. Consequently, even if the newly added action q_t is matched to the most similar action in s, by the definition of the similarity measure, the cumulative score cannot be greater than the prefix similarity (multiplied by the decay factor, following the arrival of a new action) plus the similarity of the best match (Proof omitted). We therefore use the following as our first upper-bound:

$$B_1(s) = minScore(top_k(u_{t-1}, S))\beta^2 + max_{q \in s}\{\sigma(q_t, q)\}$$

Note that since $top_k(u_{t-1}, S)$ is already computed, to calculate the bound we only need the similarity of q_t and the actions in s. As individual actions do reside in a metric space (unlike sessions/prefixes), we show in Section 5.4 how this is done efficiently.

Second bound (B_2). An alternative, less intuitive upper-bound for $\widehat{sup}^t(s)$ is achieved by dividing u_t into three disjoint subsequences w.r.t. a session s, and separately bounding the similarity to each part: (a) the segment of u_t that was already compared to s in previous iterations, denoted u_τ (b) the segment containing only the last added action q_t, and (c) the segment in between these two. By adding up the bounds for each segment we obtain a bound for the whole sequence:

Proposition 5.6.

$$\widehat{sup}^t(s) \leq max_{1 \leq j \leq |s|}\{\vec{sim}_{u_\tau, s}[j]\}\beta^{2(t-\tau)} + \frac{\beta^{2(t-\tau)} - \beta^2}{\beta^2 - 1} + \\ + max_{q \in s}\{\sigma(u_t[t], q)\}$$

The latter proposition holds for $\beta < 1$[4].

Proof Sketch. We prove by induction. We denote $s_\tau := max_{1 \leq j \leq |s|}\{\vec{sim}_{u_\tau, s}[j]\}$. The base case is when $t = \tau + 1$. We can easily show that:

$$\widehat{sup}^{\tau+1}(s) \leq s_\tau\beta^2 + max_{q \in s}\{\sigma(u_t[t], q)\}$$

We know that:

$$\widehat{sup}^{t+1}(s) \leq \widehat{sup}^t(s)\beta^2 + max_{q \in s}\{\sigma(u_{t+1}[t+1], q)\} \quad (2)$$

Assuming the inequality holds for t, we can bound $\widehat{sup}^t(s)\beta^2$:

$$\widehat{sup}^t(s)\beta^2 \leq \beta^2(s_\tau\beta^{2(t-\tau)} + \frac{\beta^{2(t-\tau)} - \beta^2}{\beta^2 - 1} + max_{q \in s}\{\sigma(u_t[t], q)\})$$

Since $max_{q \in s}\{\sigma(u_t[t], q)\} \leq 1$ we have:

$$\widehat{sup}^t(s)\beta^2 \leq s_\tau\beta^{2(t+1-\tau)} + \frac{\beta^{2(t+1-\tau)} - \beta^4 + \beta^2(\beta^2 - 1)}{\beta^2 - 1}$$

We can use the latter expression in (2) to obtain:

$$\widehat{sup}^{t+1}(s) \leq s_\tau\beta^{2(t+1-\tau)} + \frac{\beta^{2(t+1-\tau)} - \beta^2}{\beta^2 - 1} + max_{q \in s}\{\sigma(u_{t+1}[t+1], q)\}$$

Hence the inequality holds for $t + 1$ □

Importantly, note that this bound requires no action-similarity computations besides those already performed for B_1, and that all other values are either predefined constants or already computed in previous iterations. The proper upper bound $sup^t(s)$ is defined as the minimum of the two bounds: $sup^t(s) = min(B_1(s), B_2(s))$

5.4 Efficient Retrieval of Candidate Sessions

As mentioned above, we use the first bound B_1 to quickly identify relevant sessions, then compute the full bound only for the retrieved sessions, for further pruning. We use the following observation, which follows immediately from Observation 5.5.

Observation 5.7. *For $sup^t(s)$ to be not smaller than inf^t, s must contain some action q s.t.*

$$\sigma(q_t, q) \geq inf^t - minScore(top_k(u_{t-1}, S))\beta^2$$

To identify sessions that contain such actions, we employ an index structure that uses the fact that the action similarity measure defines a *metric space* (See Section 3). Specifically, in our implementation we use a *metric-tree* [8] - a popular index structure that harnesses the triangle inequality property of a metric space to facilitate a fast similarity search.

Sessions selection via the actions metric-tree. Individual actions are stored in a metric tree, with pointers from each action to the session it appears in[5]. From Observation 5.7 and our definition of the action distance notion, it follows that all sessions that satisfy the bound B_1 must contain some action q s.t.:

$$\Delta(q_t, q) \leq 1 - inf^t + minScore(top_k(u_{t-1}, S))\beta^2$$

We thus use the metric tree for a fast retrieval of all such actions q, and follow their associated pointers to identify the relevant sessions. Now we only need to compute Bound B_2 for the retrieved sessions to select those with upper bound greater than inf^t.

Example 5.8. For our running example, at $t = 5$ we retrieve candidate sessions via the metric tree, w.r.t. the lower bound 1.95 (computed as in Example 5.2). Recall that our goal is to retrieve sessions having an action q s.t. $\Delta(q_5, q) \leq 1 - inf^5 + minScore(top_1(u_4, \{\phi, \psi\}))\beta^2$, namely actions similar to q_5 = "c" with distance no more than $1 - 1.95 + 2.28 * 0.81 = 0.897$. Only

[4]For the case of $\beta = 1$, the middle part in the compound expression is defined to be 0

[5]The distance function used is the complement of our predefined action similarity function, i.e. $\Delta(q, q') = 1 - \sigma(q, q')$

the letters "C" and "c" meet this constraint, therefore sessions ψ (that contains "c") is retrieved and added to the initial candidate sessions set. Then, the upper bound for ψ is given by $sup^5(\psi) = min(\{B_1(\psi), B_2(\psi)\}) = min(2.84, 3.09) = 2.84$ (full computation omitted), which is greater than the lower bound 1.95, therefore the final set of candidate sessions is $\{\psi\}$.

5.5 Analysis of Pruning Effectiveness

Intuitively, the effectiveness of T-TopK is dependent on (1) the *amount of reduction* in the examined candidate sessions, on which the exact similarity scores are calculated, as well as on (2) the *cost of retrieving* the candidate sessions.

First, let us assess the expected time complexity of T-TopK. Let $|\widehat{C}|$ be the expected number of candidate sessions meeting the bounds (we assume $|\widehat{C}| >> k$), and let $\widehat{\tau}$ be the expected number of missing similarity vectors that need to be computed for each candidate session. Also, denote by α the cost of the metric-tree search. Thus the average time complexity of T-TopK is given by $O(\alpha + \lambda|\widehat{C}|\widehat{\tau}|\widehat{s}|)$.

The parameters $|\widehat{s}|$ (mean session size) and λ (cost of individual action similarity operation) are not controllable nor affected by the implementation of the T-TopK algorithm. We therefore analyze the performance of T-TopK according to the following parameters: α - the cost of the actions metric tree search, $|\widehat{C}|$ - the expected number of candidate sessions, and $\widehat{\tau}$ the expected number of missing vectors per candidate session.

Cost of searching the actions metric tree (α). Recall that the individual actions metric tree is used to efficiently retrieve all repository sessions that satisfy Bound B_1. Based on the cost model for metric trees proposed in [9], α is given by: $\sum_{l=1}^{L} \lambda M_{l+1} F(\widehat{r_l} + r_q)$, where: $F(x)$ is the action-distance probability distribution (i.e., the probability that the distance of two arbitrary actions $\leq x$), L is the number of levels in the tree, and M_l is the number of nodes in level l of the tree, $\widehat{r_l}$ is the mean *covering radius* of nodes at level l, and r_q is the search range (in our case, $r_q = 1 - inf^t + minScore(top_k(u_{t-1}, S))\beta^2$). Also note that M_{L+1} is the total number of individual actions stored in the metric tree. Based on this cost model, we can see that (1) naturally, increasing the size of the actions domain and the cost of action-distance computation increases α. (2) More importantly, decreasing the range r_q decreases α. The latter is greatly affected by the lower bound inf^t and the upper bound $B1$, as described next.

Expected number of candidate sessions ($|\widehat{C}|$). The number of candidate sessions directly stems from the number of sessions that satisfy both bounds $B1$ and $B2$. Intuitively, $|\widehat{C}|$ depends on three factors: (1) *How high is the lower-bound.* Naturally, the *higher* the lower-bound inf^t is, the *less* the repository sessions may surpass it. The lower bound inf^t, computed by $minScore(top_k(u_t, S_{top}^{t-1}))$ is higher, if the least similar prefix in the top-k set, computed over the sessions in S_{top}^{t-1}, is of high similarity to u_t. In turn, this holds if the probability that q_t, the newly added action to u_t will be similar enough to actions contained in the sessions of S_{top}^{t-1}, and also on the *exact* similarity threshold computed at t-1 (higher is better). (2) *How low are the upper bounds.* Similarly, *lower* upper-bounds reduce the chance that repository sessions can surpass the lower bound. Both upper bounds are lower when the decay factor β is lower (this directly stems from Observation 5.5 and Proposition 5.6). Also, for both upper bounds, if the probability that q_t is aligned to an action in $s \in S \setminus S_{top}^{t-1}$ is lower - so are the upper bounds. (3) *The underlying session similarity probability distribution* - intuitively, how many sessions, on expectation, may contain prefixes with high similarity to u_t - sufficient to satisfy Bounds $B1$ and $B2$. Naturally, if many sessions are likely to have prefixes similar to u_t, more candidates will be retrieved.

Expected number of missing similarity vectors $\widehat{\tau}$. A higher number of missing similarity vectors decreases the performance of $T - TopK$. It may happen if a session did not make it to the candidates set C for several consecutive iterations, yet is suddenly considered at time t. However, recall that many of these missing vectors may be effectively computed in user idle times. We also show in Section 6.3 that even idle-times shorter than the minimal human reaction time (250ms) are sufficient to compute a significant portion of the missing vectors.

In summary, the effectiveness of the pruning is dependent on multiple, intertwined factors that stem both from the underlying structure of the analysis sessions (and actions) as well as the problem parameters (such as the gap penalty δ and the size of k). In our experimental evaluation (See Section 6) we have empirically examined the effect of various such parameters on the performance of T-TopK. Besides using a real-life session repository, we used a multitude artificially crafted repositories, each with different underlying structure, as well as different settings of the search problem. We show that in almost all such configurations, T-TopK is highly efficient in comparison to I-TopK and surpasses other baseline algorithms by 2-3 orders of magnitude.

Additional Remarks. We conclude with two remarks. First, considering the space complexity of T-TopK - on top of storing the similarity vectors there is an additional cost induced by maintaining the metric tree. However, assuming that the metric tree is balanced, the overall space complexity of the algorithm is thus $O(|S||\widehat{s}|)$, which is the same as for I-TopK. In Section 6 we show that the additional cost induced by the metric tree is marginal, even when the session repository contains over $1.5M$ individual actions.

Last, note that in practical settings the repository is dynamic, i.e., new sessions are added and existing ones are incremented. T-TopK can be easily adapted to this setting, as newly added or incremented repository sessions merely induce more *missing similarity vectors*. Recall from Section 5.2 that missing vectors can be computed during *user idle-times* (between her consecutive actions) thereby minimizing the effect on computation time.

6 EXPERIMENTAL STUDY

As mentioned above, since the speedup achieved by our optimizations may be affected by the underlying structure of the session repositories, one would ideally like to evaluate their performance on a *multitude* of real-life session repositories with different characteristics.

However, the only publicly available repository (to our knowledge) of real-life analysis sessions is rather small [1]. Therefore, we first performed experimental evaluations on a variety of artificially crafted session repositories, each having different internal characteristics that may effect the performance of our solutions. Then, we used the session repository of [1] to verify that the same performance trends hold on real life sessions as well.

Last, let us recall that our experiments focus on running-time performance. For an evaluation of the quality of recommendations generated based on *SW-SIM* we refer the reader to, e.g., [2, 3].

In what comes next, we describe the experimental setup and the construction method for the artificial repositories in Section 6.1, then show the performance of our solution compared to numerous baseline approaches in Section 6.2. Last, we examine the scalability and performance trends w.r.t. numerous parameters of the search problem and session repositories (Section 6.3).

6.1 Experimental Datasets & Setup

Artificial Session Repositories Construction. In the absence of real-life session repositories that are large enough and publicly available, we constructed a multitude of artificial repositories, each with different underlying characteristics that may affect the performance of our solution. Each repository was constructed by first building an action (metric-) space with certain, configurable properties, then constructing repository sessions by drawing actions from this space (also w.r.t. a configurable setting). The controllable parameters and the ranges we use are depicted in Table 1.

Generating the individual action space. Recall that our algorithms model analysis actions as abstract objects in a given metric space. We represent here actions as points in an N-dimensional Euclidean space, with the Euclidean distance as the distance metric. To obtain values in $[0, 1]$ range, each element is restricted to the range $[0, \frac{1}{\sqrt{N}}]$. We the simulate a controlled degree of similarity between actions as follows: first draw a (controllable) number of action points uniformly at random (u.a.r.), to serve as cluster centers, then generate additional actions around each center using a (multi-variant) normal distribution, with the cluster-center as mean. In our experiments we varied the number N of action-space dimensions, the number of action clusters, and their radius (standard deviation), obtaining different degrees of underlying action-similarity in the repository session.

Generating repository sessions. The sessions repository is constructed in an analogous manner. We first generate a set of "seed" sessions, then use them to generate sessions with varying degrees of similarity to the seed. For each session (seed or other) we draw its length from a given normal distribution. To construct a seed-session we select (u.a.r) a sequence of actions of the given length. The rest of the repository sessions are constructed based on one of the seed-sessions. To further control the similarity of an arbitrary session s to its corresponding seed, we set a fraction p of "random" actions in s. Namely, $p \cdot l$ actions in s are randomly drawn (u.a.r) from the entire action space. The rest $(1-p) \cdot l$ actions in s are chosen from the same action-clusters as the actions in the corresponding seed session. Intuitively, setting a high value of p of "random" actions significantly decreases the potential of two sessions to be similar.

In the experiments below we used multiple repository configurations, as depicted in Table 1.

REACT-IDA Session Repository. We used the only publicly available repository (to our knowledge) of real-life sessions [1], collected as part of the experimental evaluation of an existing IDA recommender system [26]. The user sessions were performed by 56 analysts who used a web-based IDA interface in order to explore four different datasets from the cyber-security domain. The repository contains 1100 distinct actions. The average session length is 8.5 actions, and the median user idle-time is 40 seconds.

Action Similarity. Recall that both algorithms require a measure for action similarity. When using the real-life repository, we employed the similarity measure of the IDA recommender system [26] whose analysis UI was used to record the sessions.[6] For the artificial repositories, where the actions reside in a multi-dimensional metric-space, we used Euclidean distance.

Parameter	Min	Max	Default
Problem Parameters			
Decay Factor β	0.1	1	0.9
Gap Penalty δ	0.05	1	0.1
Output Size k	4	24	12
Controlled Repository Parameters			
Action Dim.	5	500	25
#Action Clusters	3K	24K	6K
Cluster Rad. (std)	0.0075	0.012	0.003
#Seed Sessions	6%	16%	10%
%Random Actions (p)	0%	100%	80%
Idle Time (s)	0	5	1
Repository Scale Parameters			
#Sessions	1K	100K	10K
Session len.	$\mathcal{N}(4, 3^2)$	$\mathcal{N}(32, 12^2)$	$\mathcal{N}(16, 3^2)$

Table 1: Problem & Repository Parameters

Default Parameters Selection. The search problem parameters (namely, the decay factor β, the gap penalty δ, and the size of k) may affect the performance of our solution. However, to capture the real-life setting where the algorithms are embedded in actual IDA recommender systems, the default values are set to optimize the *quality* of the *SW-SIM* measure, rather than the performance of our optimizations. To do so, we embedded *SW-SIM* as the top-k search component in the IDA next-step recommender system [26] (code available in [1]), and performed the hyper-parameters selection routine as described in [26]. Briefly, this is done by executing a grid search for the systems' parameters (including the top-k problem parameters), and selecting the configuration that allows the recommender system to achieve the highest qualitative performance (as determined in a predictive accuracy evaluation).

As for the artificial repository construction, intermediate values were chosen as default values, as stated in Table 1. Nevertheless, in Section 6.3 we examine the effect of varying each problem/repository parameter on the running time performance of our solution.

Hardware and Software Specifications. We implemented the algorithms presented in the previous sections in *Java 8*, using Guava (https://github.com/google/guava) for the max-heap. For the metric-tree we used an M-tree [8] Java implementation (https://github.com/erdavila/M-Tree), employing *Minimum Sum of Radii* split policy (See [8]), and a maximum node capacity of 20 objects. All experiments were conducted over *Intel Core i7-4790, 3.6GHz* machine (4 dual cores), equipped with 8GB RAM.

6.2 Baseline Comparison

We compared the performance of the T-TopK algorithm to the following baseline algorithms, each computing the exact set $top_k(u_t, S)$ according to the alignment based similarity measure (Definition 3.1). Among these, we show the results of the following baselines: (1) **Naive Sequential Search (NSS)** that retrieves the set of similar prefixes by iteratively comparing u_t to each prefix of each repository session in S using a direct implementation of Definition 3.1 with no further optimizations, then selects

[6] The similarity notion considers both the action syntax and (a signature of) the results. (See [26]).

Baseline	REACT-IDA (1.1K actions)		Repo-10 (160K actions)		Repo-100 (1.6M actions)	
	Time (ms)	#ops	Time (ms)	#ops	Time (s)	#ops
NM-Tree	42.3	27.5K	18,503	12.3M	186	123M
CSE	55	36.6K	18,224	12.1M	183	121M
NSS	51.9	35K	17,204	11.5M	170	123M
OSS	7.6	5.1K	3,242	1.36M	32	13.6M
I-TopK	1	1.1K	237	160K	2.4	1.6M
T-TopK	**0.5**	**641**	**59**	**39K**	**0.9**	**722K**

Table 2: Baseline Comparison. We compared various baseline algorithms in terms of running times and the number of similarity operations

the top-k similar prefixes. (2) **Optimized Sequential Search (OSS)**, which employs the first optimization described in Section 4, instead of computing the alignment matrix for each prefix. Namely, it iterates over all sessions in $s \in S$, constructs a single alignment matrix $A_{u_t,s}$, and uses Observation 4.1 to derive the similarity scores of u_t and each prefix of s. (3) **I-TopK**, the iterative algorithm depicted in Section 5.1 which employs both of the optimizations in Section 4. (4) **T-TopK**, the optimized algorithm described in Sections 5.2-5.4. (5) **Constant Shift Embedding (CSE)[31]** and (6) **NM-Tree [34]**, are general purpose solutions for top-k search in a non-metric space. Both use *metrization* techniques: CSE use a simple solution that increments each distance score by a predefined constant, so the triangle inequality is enforced. Then, all session prefixes in S are stored in a *metric tree* (w.r.t. the new metric space). When given the ongoing session u_t, the metric tree is traversed, using the new metric to obtain the exact set of top-k similar prefixes. NM-tree uses a more sophisticated similarity-preserving transformation on the original (non-metric) distance measure, then employs an extension of the metric tree to index and query the transformed metric space.

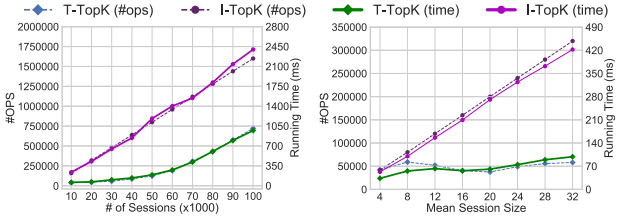
Finally, recall from Section 2 that optimization techniques dedicated to other similarity measures (e.g., DTW, Global Sequence Alignment) are inadequate for *SW-SIM*, therefore could not serve as baselines. Also, approximation-based solutions were omitted as well since this work concerns with the case of exact computation of the similarity scores.

Evaluation Process. We evaluated all baselines using a multitude of configurations for the top-k search problem (i.e. the constants β, δ of the similarity measure, and k), on various configurations of artificial repositories.

The evaluation process is as follows. Given a session repository S, in each trial we draw a random session prefix as u_t, then employ each baseline to retrieve $top_k(u_t, S)$. We performed 100 trials for each repository, capturing the average execution time and memory consumption, and the average number of action similarity operations performed in each run.

Results. Table 2 presents a representative sample of the results, showing the average running time and the number of action similarity operations (denoted #ops) obtained by each baseline. The comparison is presented for the REACT-IDA repository (Containing 1.1K actions) as well as Repo-10 and Repo-100, which are two artificial session repositories (with the default configuration stated in Table 1) containing 10K and 100K sessions (resp., 160K and 1.6M actions).

First, for both Repo-10 and Repo-100, the performance of *NM-Tree* and *CSE* is almost on-par with NSS (the naive sequential search). This could be due to the fact that transforming the non-metric space may induce high overlap between the metric-tree nodes, therefore the search deteriorates to sequential search plus additional overhead induced by the metric tree (See [34]). Second, note that OSS improves running times by 5X (and #ops by 9X)



(a) Repository Size **(b) Mean Session len.**

Figure 3: Effect of Scale Parameters

over NSS due to its efficient use of Observation 4.1. However, a more significant improvement of 15X in running times (and 9X in terms of #ops) is achieved by I-TopK which utilizes the incremental computation. **Finally, an additional 2-4X speedup is obtained by T-TopK, resulting in an overall improvement of 189X to 291X over the NSS baseline**.

The performance trends on the REACT-IDA repository (comprising the smaller collection of real-life sessions) are similar, with some minor variation. We can first see that NM-Tree performs slightly better than NSS and CSE. This is due to the underlying structure of the dataset that allows the NM-Tree to perform (and store) the distance-preserving transformations more efficiently.

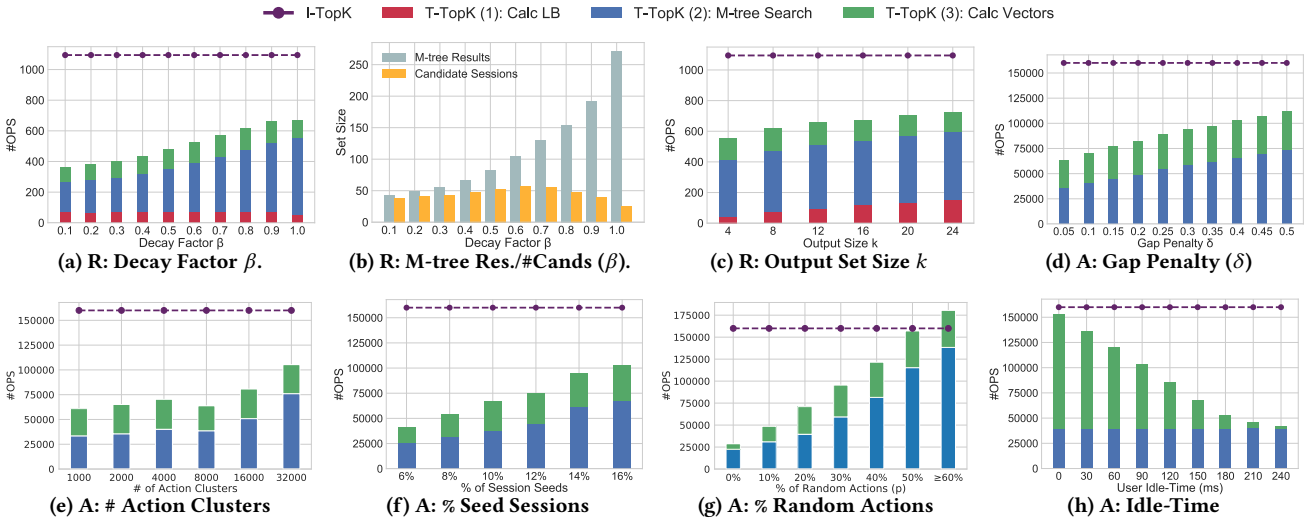
However, both our optimized algorithms perform significantly better, with T-TopK dominating. Interestingly, while the REACT-IDA is fairly small, the pruning-based optimizations of T-TopK are still highly effective. T-TopK retains a 2X speedup compared to I-TopK, and significantly outperforms the other baselines.

Next, we present a performance comparison of the algorithms when employed on a multitude of different repositories and with different problem parameters, as well as an examination of the computation segments in T-TopK. Since in all configurations, the performance of T-TopK was significantly better (by at least an order of magnitude) than the baseline algorithms, we omit them from presentation and use only I-TopK as a baseline.

6.3 Scalability & Parameters Effect

We next analyze the performance of the T-TopK Algorithm compared to I-TopK, by varying the problem parameters as well as the repository parameters, one at a time, while keeping the rest at their default values (As in Table 1). We measured both the running times, number of action similarity operations (*#ops*), and memory consumption.

Scalability Parameters. Figure 3a depicts the performance of the algorithms when increasing the number of sessions in the repository. For both algorithms we can see a rather linear increase in running times (correspondingly, *#ops* however the two plots overlap), but T-TopK shows a significant speed up of 3X (on average) over I-TopK. Moreover, as we can see in Figure 3b, when the mean *session length* increases, T-TopK performance stays stable compared to a linear increase for I-TopK. This demonstrates that



(a) R: Decay Factor β. (b) R: M-tree Res./#Cands (β). (c) R: Output Set Size k (d) A: Gap Penalty (δ)

(e) A: # Action Clusters (f) A: % Seed Sessions (g) A: % Random Actions (h) A: Idle-Time

Figure 4: Parameters Effect on REACT-IDA (R) and Artificial (A) Datasets

the effectiveness of the threshold-based approach further grows for longer sessions. As for memory consumption, the maximal usage for T-TopK did not exceed 74*MB* even for a repository as big as 100K sessions (1.6M individual actions), which is negligible in practice. We therefore do not further discuss memory consumption in the next experiments.

Problem Parameters. Recall that the problem parameters are the gap penalty δ, the decay factor β and the output set size k. We examine their effect on the performance, when varying each parameter, and keeping the rest at their default values. To gain a better insight on the computation of T-TopK, we break the total *#ops* performed into three computational segments: (1) forming the initial similarity lower bound, (2) the metric-tree search, and (3) computing similarity vectors for candidate sessions. The corresponding running time trends are similar, therefore omitted from the figures below. Also, as Segment (3) is negligibly small when using the default idle-time, we further restrict the default user idle-time to 150ms, which naturally stresses T-TopK. The performance of I-TopK in terms of #ops is represented by a dotted flat line since it is not dependent on the problem parameters.

We observe that increasing each of the problem parameters results in a minor increase in #ops for T-TopK. Figure 4a and Figure 4c show the effect of the decay factor β and the output size k on performance, when the algorithms are employed on the REACT-IDA repository, and Figure 4d shows the effect of the gap penalty δ on the default artificial repository (the effect of δ on the REACT-IDA repository was marginal, therefore the figure is omitted). The increase in performance is expected, since the values of the problem parameters play a part in the lower/upper bound computations: (1) The decay factor β affects the upper bound, therefore lower values induce more restrictive candidate selection, and thereby better performance. (2) Increasing the output size k causes a decrease in the lower bound threshold inf^t, thus more candidate sessions are examined.

To gain a deeper insight on the performance w.r.t. the computation segments, we examine in Figure 4b, the number of sessions retrieved in the metric-tree search results (i.e. sessions satisfying bound B_1) and the number of sessions among them that also meet bound B_2 (hence satisfy the upper bound), for varying β values. We can see that the first set grows with β (hence the cost of Segment 2 increases), but many sessions are pruned via bound B_2 (thus the cost of Segment 3 does not increase).

Repository Parameters. As is the case for most top-k search optimizations and dedicated data structures, the underlying structure of the data points may impact the performance of the T-TopK algorithm (note that the incremental similarity optimizations presented in Section 4 are not affected by such parameters).

Therefore, to properly evaluate the effectiveness of T-TopK we constructed a multitude of artificial repositories (as described above), each with a different underlying structure that stems from a particular repository-parameters configuration. We varied each construction parameter and examined its effect on the performance of T-TopK (keeping the rest of the parameters at their default configuration). Figures 4e, 4f and 4g depict the performance of T-TopK compared to I-TopK when varying the amount of individual action clusters, percentage of "seed sessions", and the percentage of random actions in a session, respectively. Intuitively, increasing the values of each of these parameters inflict more "randomness" on the underlying structure of the data points (sessions), therefore the mean similarity score of two arbitrary sessions decreases. In turn, lower similarity scores imply smaller lower bounds, therefore more candidate sessions are examined. With the exception of one case, in all observed settings, even when increasing these parameters, T-TopK was still effective, outperforming I-TopK. The only setting where I-TopK outperformed T-TopK was when setting a high (> 60%) percentage of random actions in the repository sessions, which significantly decreases the average session similarity score (Importantly, further increasing this parameter had a negligible effect on performance). However, as we show next, given slightly higher user idle-times, (which is a very reasonable assumption) T-TopK still outperforms I-TopK, even when the percentage of random actions is high, as the computation segment of calculating similarity vectors (in green) is significantly diminished.

User Idle-Times. We measured how the mean user idle-time affects the performance of T-TopK. Such idle time between consecutive actions of the same user often takes several seconds. Figure 4h depicts the performance when varying the expected time ranges from 0 to 0.24 seconds. Interestingly, even the latter, which is lower than the minimal human reaction time to a visual stimulus, was already sufficient to compute all missing vectors offline. As expected, T-TopK improves when the idle-time increases, and when all missing vectors are computed offline - T-TopK obtains almost a 3X speedup over I-TopK.

7 CONCLUSION

In this work, we show (for the first time, to our knowledge) that by utilizing the progressive nature of IDA sessions, a major running-time speedup (of over $180X$) can be obtained, compared to current solutions for top-k similarity search of analysis sessions. Our solution allows IDA recommendation systems to effectively rely on much larger session repositories while retaining interactive response times.

However, the scope of our work is limited in two main aspects: First, our solution is dedicated to the *SW-SIM* similarity measure for analysis sessions. While considered a very comprehensive, useful measure, some IDA recommendation systems use different session similarity measures. Nevertheless, we believe that the core principles of our solution may be applicable (with some tweaking and adaptations) to other similarity measures - even outside the scope of IDA, (e.g. scientific/business workflows, prescriptive analytics, mashups [5, 13, 42]).

Second, as common in many filter-and-refine based frameworks, the efficiency of the pruning techniques used by the T-TopK algorithm may be affected by underlying characteristics of the session repository, as well as the parameters of the search problem. In the absence of large enough, publicly available real-life IDA workloads, our experimental evaluation is performed, primarily, over a multitude of carefully crafted artificial repositories, each with different characteristics. Although we have shown that similar performance trends occur on the real-life session repository as well (the only one that is publicly available), it is still left to demonstrate the performance speedup on other real-life IDA workloads (when such become publicly available). Nevertheless, recall that the efficiency of the single-alignment optimizations, and the computation of the similarity vectors (as presented in Section 4), is independent of the session repository or problem parameters - and even these alone still provide a significant speedup of $40 - 80X$ compared to the currently available solutions.

Acknowledgments. This work has been partially funded by the Israel Innovation Authority (MDM), the Israel Science Foundation, the Binational US-Israel Science foundation, Len Blavatnik and the Blavatnik Family foundation, and Intel® AI DevCloud.

REFERENCES

[1] [n. d.]. REACT: IDA Benchmark Dataset. ([n. d.]). https://git.io/fhyOR.
[2] Julien Aligon, Enrico Gallinucci, Matteo Golfarelli, Patrick Marcel, and Stefano Rizzi. 2015. A collaborative filtering approach for recommending OLAP sessions. *DSS* 69 (2015).
[3] Julien Aligon, Matteo Golfarelli, Patrick Marcel, Stefano Rizzi, and Elisa Turricchia. 2014. Similarity measures for OLAP sessions. *KAIS* 39 (2014).
[4] Marie Aufaure, Nicolas Kuchmann-Beauger, Patrick Marcel, Stefano Rizzi, and Yves Vanrompay. 2013. Predicting your next OLAP query based on recent analytical sessions. *DaWaK* (2013).
[5] Ralph Bergmann and Yolanda Gil. 2014. Similarity assessment and efficient retrieval of semantic workflows. *Information Systems* 40 (2014), 115–127.
[6] Michael W Berry and Murray Browne. 2005. *Understanding search engines: mathematical modeling and text retrieval*. Vol. 17. Siam.
[7] Paolo Ciaccia and Marco Patella. 2002. Searching in metric spaces with user-defined and approximate distances. *ACM Transactions on Database Systems (TODS)* 27, 4 (2002), 398–437.
[8] Paolo Ciaccia, Marco Patella, and Pavel Zezula. 1997. M-tree: An Efficient Access Method for Similarity Search in Metric Spaces. In *VLDB*.
[9] Paolo Ciaccia, Marco Patella, and Pavel Zezula. 1998. A cost model for similarity queries in metric spaces. In *SIGCART*. ACM.
[10] Dong Deng, Guoliang Li, He Wen, HV Jagadish, and Jianhua Feng. 2016. META: an efficient matching-based method for error-tolerant autocompletion. *PVLDB* 9, 10 (2016).
[11] Michel Marie Deza and Elena Deza. 2009. Encyclopedia of distances. Springer, 1–583.
[12] Remco Dijkman, Marlon Dumas, Boudewijn Van Dongen, Reina Käärik, and Jan Mendling. 2011. Similarity of business process models: Metrics and evaluation. *Information Systems* 36, 2 (2011), 498–516.
[13] Fan Du, Catherine Plaisant, Neil Spring, and Ben Shneiderman. 2016. EventAction: Visual analytics for temporal event sequence recommendation. In *2016 IEEE Conference on Visual Analytics Science and Technology (VAST)*. IEEE, 61–70.
[14] Robert C Edgar. 2010. Search and clustering orders of magnitude faster than BLAST. *Bioinformatics* 26, 19 (2010), 2460–2461.
[15] Magdalini Eirinaki, Suju Abraham, Neoklis Polyzotis, and Naushin Shaikh. 2014. Querie: Collaborative database exploration. *TKDE* (2014).
[16] Arnaud Giacometti, Patrick Marcel, and Elsa Negre. 2009. *Recommending multidimensional queries*. Springer Berlin Heidelberg.
[17] Shengyue Ji, Guoliang Li, Chen Li, and Jianhua Feng. 2009. Efficient interactive fuzzy keyword search. In *Proceedings of the 18th international conference on World wide web*. ACM, 371–380.
[18] W James Kent. 2002. BLAT—the BLAST-like alignment tool. *Genome research* 12, 4 (2002), 656–664.
[19] Eamonn Keogh and Chotirat Ann Ratanamahatana. 2005. Exact indexing of dynamic time warping. *Knowledge and information systems* 7, 3 (2005), 358–386.
[20] Sung-Ryul Kim and Kunsoo Park. 2000. A dynamic distance table. *CPM* (2000).
[21] Gad M Landau, Eugene W Myers, and Jeanette P Schmidt. 1998. Incremental string comparison. *SIAM J. Comput.* 27, 2 (1998), 557–582.
[22] Ben Langmead, Cole Trapnell, Mihai Pop, and Steven Salzberg. 2009. Ultrafast and memory-efficient alignment of short DNA sequences to the human genome. *Genome biology* (2009).
[23] David Leake and Joseph Kendall-Morwick. 2008. Towards case-based support for e-science workflow generation by mining provenance. In *European Conference on Case-Based Reasoning*. Springer, 269–283.
[24] Guoliang Li, Jiannan Wang, Chen Li, and Jianhua Feng. 2012. Supporting efficient top-k queries in type-ahead search. In *Proceedings of the 35th international ACM SIGIR conference on Research and development in information retrieval*. ACM, 355–364.
[25] Elaine R Mardis. 2008. The impact of next-generation sequencing technology on genetics. *Trends in genetics* 24, 3 (2008), 133–141.
[26] Tova Milo and Amit Somech. 2018. Next-Step Suggestions for Modern Interactive Data Analysis Platforms. In *KDD*.
[27] David Novak and Pavel Zezula. 2006. M-Chord: a scalable distributed similarity search structure. In *ICSIS*.
[28] Panagiotis Papapetrou, Vassilis Athitsos, George Kollios, and Dimitrios Gunopulos. 2009. Reference-based alignment in large sequence databases. *VLDB* (2009).
[29] Srinivasan Parthasarathy, Mohammed Javeed Zaki, Mitsunori Ogihara, and Sandhya Dwarkadas. 1999. Incremental and interactive sequence mining. In *CIKM*.
[30] Stefano Rizzi and Enrico Gallinucci. 2014. CubeLoad: a parametric generator of realistic OLAP workloads. In *CAISE*.
[31] Volker Roth, Julian Laub, Motoaki Kawanabe, and Joachim M Buhmann. 2003. Optimal cluster preserving embedding of nonmetric proximity data. *TPAMI* (2003).
[32] Hiroaki Sakoe and Seibi Chiba. 1978. Dynamic programming algorithm optimization for spoken word recognition. *TASSP* (1978).
[33] Victor Sepulveda and Benjamin Bustos. 2010. CP-Index: using clustering and pivots for indexing non-metric spaces. In *SISAP*.
[34] Tomáš Skopal and Jakub Lokoč. 2008. NM-tree: Flexible approximate similarity search in metric and non-metric spaces. In *DEXA*.
[35] Temple F Smith and Michael S Waterman. 1981. Identification of common molecular subsequences. *Journal of molecular biology* 147, 1 (1981), 195–197.
[36] Johannes Starlinger, Sarah Cohen-Boulakia, Sanjeev Khanna, Susan B Davidson, and Ulf Leser. 2016. Effective and efficient similarity search in scientific workflow repositories. *FGCS* 56 (2016), 584–594.
[37] Narayanan Sundaram, Aizana Turmukhametova, Nadathur Satish, et al. 2013. Streaming similarity search over one billion tweets using parallel locality-sensitive hashing. *VLDB* (2013).
[38] Yu Tang, Yilun Cai, Nikos Mamoulis, Reynold Cheng, et al. 2013. Earth mover's distance based similarity search at scale. *VLDB* (2013).
[39] Tatiana A Tatusova and Thomas L Madden. 1999. BLAST 2 Sequences, a new tool for comparing protein and nucleotide sequences. *FEMS Microbiol. Lett.* 174, 2 (1999).
[40] Sebastian Wandelt, Johannes Starlinger, Marc Bux, and Ulf Leser. 2013. RCSI: Scalable similarity search in thousand (s) of genomes. *VLDB* (2013).
[41] Ting Xie, Varun Chandola, and Oliver Kennedy. 2018. Query log compression for workload analytics. *Proceedings of the VLDB Endowment* 12, 3 (2018), 183–196.
[42] Sen Yang, Xin Dong, Leilei Sun, Yichen Zhou, Richard A Farneth, Hui Xiong, Randall S Burd, and Ivan Marsic. 2017. A Data-driven Process Recommender Framework. In *KDD*.
[43] Xiaoyan Yang, Cecilia M Procopiuc, and Divesh Srivastava. 2009. Recommending join queries via query log analysis. In *ICDE*.
[44] Youwei Yuan, Weixin Chen, Guangjie Han, and Gangyong Jia. 2017. OLAP4R: A Top-K Recommendation System for OLAP Sessions. *TIIS* (2017).