A framework for routing and spectrum assignment in optical networks, driven by combinatorial properties

Pedro H. Fernandes da Silva
LIMOS, Clermont Auvergne Institute of Technology,
Clermont-Ferrand and IMT Atlantique, Brest, France
pedrohenrique.phfs@gmail.com

Hervé Kerivin
LIMOS, Clermont Auvergne Institute of Technology
Clermont Auvergne University, Clermont-Ferrand, France
herve.kerivin@limos.fr

Juan Pablo Nant
LIMOS, Clermont Auvergne Institute of Technology
Clermont Auvergne University, Clermont-Ferrand, France
jpnant@gmail.com

Annegret K. Wagler
LIMOS, Clermont Auvergne Institute of Technology
Clermont Auvergne University, Clermont-Ferrand, France
annegret.wagler@limos.fr

ABSTRACT

The routing and spectrum assignment problem is an NP-hard problem that has received increasing attention during the last years. The majority of existing models for the problem uses edge-path formulations where variables are associated with all possible routing paths so that the number of variables grows exponentially with the size of the instance. To bypass this difficulty, precomputed subsets of all possible paths per demand are typically used, which cannot guarantee optimality of the solutions in general. Our contribution is to provide a framework for the use of edge-path formulations to minimize the spectrum width of a solution. For that, we select an appropriate subset of paths to operate on with the help of combinatorial properties in such a way that optimality of the solution can be guaranteed. Computational results indicate that our approach is indeed promising to solve the routing and spectrum assignment problem.

1 INTRODUCTION

Optical networks represent a crucial infrastructure for our information society and use light as a communication medium between sending and receiving nodes. For over two decades, *Wavelength-Division Multiplexing (WDM)* has been the most popular technology used in fiber-optic communications. WDM combines multiple wavelengths to simultaneously transport signals over a single optical fiber, but has to select the wavelengths from a rather coarse fixed grid of frequencies specified by the International Telecommunication Union (ITU) and leads to an inefficient use of spectral resources. In response to the sustained growth of data traffic volumes in communication networks, so-called flexgrid optical networks have been introduced to enhance the spectrum efficiency and enlarge the network capacity. In such networks, the frequency spectrum of an optical fiber is divided into narrow frequency slots and any sequence of consecutive slots can form a channel on optical fibers to create an optical connection, called *lightpath*, and thus enables capacity gain by allocating minimum required bandwidth [8].

The *Routing and Spectrum Assignment (RSA)* problem consists of establishing the lightpaths for a set of traffic demands, given as sending and receiving nodes and frequency slot numbers. Since lightpaths are determined by a route and a channel, the RSA problem involves finding a route and assigning a channel of frequency slots for each demand. To comply with ITU recommendation, the following constraints need to be respected:

- *slot continuity*: the slots remain the same on all the links of a route;
- *slot contiguity*: the slots allocated to a demand must be contiguous;
- *non-overlapping slot*: on each link, a slot can be allocated to at most one demand.

More precisely, we are given an optical network $G = (V, E)$ with edge length l_e for all $e \in E$, an optical spectrum $S = \{1, \ldots, \bar{s}\}$, and a set $\mathcal{D}$ of demands between pairs o_k, d_k of nodes in G specifying the maximum length $\bar{l}_k$ of a route and the number w_k of required slots. The routing selects, for each demand $k = (o_k, d_k, \bar{l}_k, w_k) \in \mathcal{D}$, an (o_k, d_k)-path P_k of length at most $\bar{l}_k$ as route from o_k to d_k through G. The spectrum assignment consists of selecting, for each $k \in \mathcal{D}$, a channel $S_k \subseteq S$ of w_k consecutive frequency slots that satisfies the three above constraints. We denote a routing of $\mathcal{D}$ by $\mathcal{P} = \{P_k : k \in \mathcal{D}\}$, and a spectrum assignment by $\mathcal{S} = \{S_k : k \in \mathcal{D}\}$ so that any pair $(\mathcal{P}, \mathcal{S})$ is a solution to the RSA problem.

In addition, the selected set of lightpaths is supposed to minimize a chosen objective function, e.g. minimize the number of edges in the routing paths P_k [19], minimize the number of edges from the network used to route the demands [17], or minimize the spectrum width (and, thus, the width of the subspectrum of S used for the spectrum assignment) [2].

The RSA problem has been shown to be NP-hard [3, 18]. In fact, if all routes are already known or uniquely determined (e.g. if the optical network is a tree), then the RSA problem reduces to the spectrum assignment and only consists of determining the demand's channels. It is NP-complete to decide whether there is a feasible spectrum assignment within a given optical spectrum, even if the optical network is a path, see e.g. [16]. This makes the RSA problem much harder than the WDM problem which is polynomially solvable on paths, see e.g. [5].

To solve the RSA problem, various approaches have been studied in the literature, based on different Integer Linear Programming (ILP) models. Few models use *edge-node formulations* which are compact in terms of the number of variables and constraints, see e.g. [2, 17, 19] and [1] for an overview, but have the disadvantage that the routing is rather involved. As noticed in [7], the models from [2, 17, 19] are incomplete as their feasible region is a superset of all feasible solutions to the RSA problem. The first complete edge-node formulation presented in [7] exactly encodes the feasible solutions, but requires an exponential number of constraints.

The majority of the existing models uses an *edge-path formulation* where for each demand, variables are associated with all possible routes for this demand, leading to an exponential number of variables issued from the total number of all feasible paths between origin-destination pairs in the network, which grows exponentially with the size of the network. To bypass the exponential number of variables, edge-path formulations with a restricted precomputed subset of all possible paths per demand have been studied, e.g. in [10, 12, 17, 19], see [19] for an overview. However, such formulations cannot guarantee optimality of the solutions in general (as only a subset of paths is considered and, thus, a restricted problem is solved). In order to find optimal solutions to the RSA problem w.r.t. any objective function with the help of an edge-path formulation, all possible paths have to be taken into account. As the explicit models are far too big for computation, it is in order to apply column-generation methods. However, computational results from [11, 13, 15] show that the size of the instances that can be solved that way is rather limited[1].

Our goal is to compute the minimum spectrum width which has turned out to be particularly difficult, see e.g. [2, 7]. For that, we provide a framework for the use of edge-path formulations that selects an appropriate subset of paths to operate on with the help of combinatorial properties in such a way, that it is neither necessary to enumerate all possible routing paths nor to apply generic column generation techniques, but that optimality of the solution can still be guaranteed.

The idea is to iterate two major steps: on the one hand, a min-cost multi-commodity flow computes a lower bound on the minimum spectrum width and provides us with routing paths, on the other hand, solving an edge-path formulation using the already generated subset of routing paths provides a solution and an upper bound on the minimum spectrum width. As long as there is a gap between the lower and upper bounds, we add constraints to forbid (partial or full) routings that cause the use of a larger spectrum than the current lower bound, and iterate both major steps until the lower bound equals the upper bound and, thus, an optimal solution has been found (or infeasibility of the instance has been detected).

In the following, we present details on all steps involved in the framework in Section 2, we provide computational results in Section 3, and close with some concluding remarks and lines of future research.

[1]An exception is an edge-path formulation from [4] that seems to be scalable to real-size instances by using column-generation methods. However, the authors of [4] consider an asymmetric version of the RSA problem where each link of the optical network is composed by two optical fibers to be used to transmit signals in one direction only. This makes the spectrum assignment easier (as less restrictions have to be taken into account), but is not used very often in practice by network operators as that way it is not possible to use the full spectral resources of the optical links.

2 FRAMEWORK TO COMPUTE THE MINIMUM SPECTRUM WIDTH

In order to compute the minimum spectrum width, we adopt a reinterpretation of the spectrum assignment as interval coloring of the edge intersection graph $I(\mathcal{P})$ of the routing $\mathcal{P}$ [9]. Each path $P_k \in \mathcal{P}$ becomes a node of $I(\mathcal{P})$, two nodes are joined by an edge if their corresponding paths in G are in conflict as they share an edge. An interval coloring in $I(\mathcal{P})$ corresponds to the spectrum assignment: assign a frequency interval S_k of w_k consecutive frequency slots (*slot contiguity*) to every node k and, thus, to every path P_k (*slot continuity*) such that the intervals of adjacent nodes are disjoint (*non-overlapping slots*).

Let $\mathbf{w} \in \mathbf{Z}_+^{|\mathcal{D}|}$ be the vector whose entries w_k are the slot requirements associated with the demands $k \in \mathcal{D}$. The interval chromatic number $\chi_I(I(\mathcal{P}), \mathbf{w})$ is the smallest size of a spectrum such that $I(\mathcal{P})$ weighted with w_k for each path P_k has a proper interval coloring. Given G and $\mathcal{D}$, the minimum spectrum width of any solution to the RSA problem thus equals

$$\chi_I(G, \mathcal{D}) = \min\{\chi_I(I(\mathcal{P}), \mathbf{w}) : \mathcal{P} \in \mathcal{R}\}$$

where $\mathcal{R}$ denotes the set of all possible routings of the demands $\mathcal{D}$ in G. Our goal is to compute $\chi_I(G, \mathcal{D})$ which has turned out to be particularly difficult, see e.g. [2, 7].

Lower bounds on $\chi_I(G, \mathcal{D})$. Consider the following two lower bounds of $\chi_I(G, \mathcal{D})$ that are exclusively related to the routing aspect of the problem (not yet taking the spectrum assignment into account).

We denote by $\ell(G, \mathcal{D})$ the minimum number of slots that need to be installed on all edges of the optical network G to allow a routing of all demands in $\mathcal{D}$. The value $\ell(G, \mathcal{D})$ corresponds to the maximum edge load $w(\mathcal{P}) = \max\{\sum_{P_k \ni e} w_k : e \in E\}$ in the most balanced routing $\mathcal{P}$, i.e., to the minimum maximum edge load, taken over all possible routings: we call

$$\ell(G, \mathcal{D}) = \min\{w(\mathcal{P}) : \mathcal{P} \in \mathcal{R}\}$$

the *load bound.* Due to the non-overlapping slot condition, all channels S_k of paths routed along a same edge of G need to be disjoint, thus, $\ell(G, \mathcal{D})$ is a lower bound of $\chi_I(G, \mathcal{D})$.

We further consider the weighted clique number $\omega(I(\mathcal{P}), \mathbf{w})$ of the edge intersection graph $I(\mathcal{P})$ of the routing $\mathcal{P}$ (that is the maximum weight of a clique, a set of pairwise adjacent nodes, in $I(\mathcal{P})$, taking the node weights $\mathbf{w}$ into account). We denote by

$$\omega(G, \mathcal{D}) = \min\{\omega(I(\mathcal{P}), \mathbf{w}) : \mathcal{P} \in \mathcal{R}\}$$

the *clique bound*, i.e., the minimum over all maximum weighted cliques in $I(\mathcal{P})$, taken over all possible routings $\mathcal{P}$. On the one hand, all paths in a routing $\mathcal{P}$ passing through a same edge e of G are mutually in conflict and form a clique in $I(\mathcal{P})$, which shows that $\ell(G, \mathcal{D})$ is a lower bound of $\omega(G, \mathcal{D})$. On the other hand, all channels S_k of paths P_k forcing a clique in $I(\mathcal{P})$ need to be disjoint due to the non-overlapping slot condition such that $\omega(I(\mathcal{P}), \mathbf{w}) \leq \chi_I(I(\mathcal{P}), \mathbf{w})$ holds for any $I(\mathcal{P})$ and, thus, $\omega(G, \mathcal{D})$ is a lower bound of $\chi_I(G, \mathcal{D})$:

$$\ell(G, \mathcal{D}) \leq \omega(G, \mathcal{D}) \leq \chi_I(G, \mathcal{D}). \quad (1)$$

There are instances of the RSA problem where there is a gap between any two parameters from this chain, see Exp. 2.1.

Example 2.1. Consider the following instance of the RSA problem with the optical network G shown in Fig. 1 and the following set $\mathcal{D}$ of demands:

k	$o_k \to d_k$	$\bar{l}_k$	w_k	path P_k	channel S_k
1	$a \to c$	3	1	$a \to b \to c$	3
2	$c \to e$	3	2	$c \to b \to d \to e$	1 2
3	$e \to f$	3	2	$e \to d \to f$	3 4
4	$f \to g$	3	2	$f \to d \to g$	1 2
5	$g \to h$	3	2	$g \to d \to h$	3 4
6	$h \to a$	3	2	$h \to d \to b \to a$	5 6

As the network G is a tree, there is a unique routing $\mathcal{P}$, as indicated in the table above.

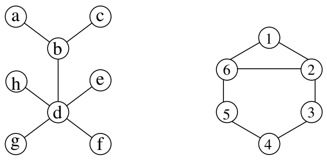

Figure 1: A network G and $I(\mathcal{P})$ of the routing.

Since the load of all edges incident to node d equals 4, $\ell(G, \mathcal{D}) = 4$ follows. The edge intersection graph $I(\mathcal{P})$ of the routing is also shown in Fig. 1. The nodes 1, 2, 6 form a clique of weight 5, hence we have $\omega(G, \mathcal{D}) = 5$. Any interval coloring of $I(\mathcal{P})$ needs at least 6 colors, as indicated in the table above. Hence, there is a gap between any two parameters from the chain (1).

We call a clique Q in $I(\mathcal{P})$ a *non-edge clique* if the routing paths composing Q pairwise intersect, but do not all meet in a same edge (like the clique formed by nodes 1, 2, 6 in the above example). Only non-edge cliques can cause a gap between $\ell(G, \mathcal{D})$ and $\omega(G, \mathcal{D})$.

A graph is *superperfect* if and only if the weighted clique number and the interval chromatic number coincide for all possible non-negative integral node weights, see e.g. [6]. Only non-superperfect subgraphs of $I(\mathcal{P})$ can cause $\omega(I(\mathcal{P}), \mathbf{w}) < \chi_I(I(\mathcal{P}), \mathbf{w})$ for the given weight $\mathbf{w}$ (as the 5-hole formed by nodes 2, 3, 4, 5, 6 in the above example), and, thus a gap between $\omega(G, \mathcal{D})$ and $\chi_I(G, \mathcal{D})$.

We here restrict to the search for cliques as it has been shown in [9] that there are too many different non-superperfect subgraphs that may occur in $I(\mathcal{P})$ and thus, it is questionable whether the time spent for their analysis is a gain for the overal running time.

Multi-commodity flows (MCF). We use multi-commodity flows in an auxiliary network G_f constructed from G to handle the lower bound and to determine routings $\mathcal{P}$.

We denote by $G_f = (V, A)$ the directed graph obtained from the optical network $G = (V, E)$ by replacing every edge $e = uv$ of G by a pair of oppositely-directed arcs $a = (u, v), \bar{a} = (v, u)$. We say that $a = (u, v)$ is the arc *outgoing* from u and *incoming* to v and denote by $\delta^-(v)$ the set of arcs incoming to v and by $\delta^+(v)$ the set of arcs outgoing from v.

Each demand $k \in \mathcal{D}$ corresponds to a commodity f_k with source $o_k \in V$ and sink $d_k \in V$ in G_f. The ILP to determine the minimum number cap of slots needed on all edges of G to allow the routing of all demands $k \in \mathcal{D}$ is as follows:

$$\begin{array}{lll}
\min\ cap & & \\
\sum_{a \in \delta^+(o_k)} f_k(a) & = 1\ \forall k & \\
\sum_{a \in \delta^-(o_k)} f_k(a) & = 0\ \forall k & \\
\sum_{a \in \delta^-(d_k)} f_k(a) & = 1\ \forall k & \\
\sum_{a \in \delta^+(d_k)} f_k(a) & = 0\ \forall k & \\
\sum_{a \in \delta^-(v)} f_k(a) - \sum_{a \in \delta^+(v)} f_k(a) & = 0\ \forall k, \forall v \neq o_k, d_k & (2)\\
\sum_{a \in \delta^-(v)} f_k(a) & \leq 1\ \forall k, \forall v \neq o_k, d_k & \\
\sum_{a \in \delta^-(v)} l_a f_k(a) & \leq \bar{l}_k\ \forall k & \\
\sum_{k \in \mathcal{D}} w_k f_k(a) + \sum_{k \in \mathcal{D}} w_k f_k(\bar{a}) & \leq cap\ \forall \text{ pairs } a, \bar{a} \in A & \\
f_k(a) & \in \{0, 1\} \forall k, a &
\end{array}$$

As the sum of flow values on each pair of oppositely-directed arcs $a = (u, v), \bar{a} = (v, u)$ (and, thus, on each edge $e = uv$ of G) is bounded by cap and the value of cap is minimized, ILP (2) indeed computes the load bound by

$$\ell(G, \mathcal{D}) = cap.$$

This happens in the initial run of the multi-commodity flow. If $\ell(G, \mathcal{D}) > \bar{s}$, the considered instance $(G, \bar{s}, \mathcal{D})$ is infeasible. Otherwise, the computed multi-commodity flow provides us with a routing $\mathcal{P}$ which allows us to continue with solving an edge-path formulation.

In later runs of the multi-commodity flow, we have to take forbidden cliques Q (of weight $w(Q) > \ell(G, \mathcal{D})$) and forbidden routings (to never consider a same routing twice) into account. For that, ILP (2) is enhanced by the following constraints: forbidden routing constraints associated with a routing $\mathcal{P}$

$$\sum_{k \in \mathcal{D}} \sum_{a \in A^k_{\mathcal{P}}} f_k(a) \leq \sum_{k \in \mathcal{D}} |A^k_{\mathcal{P}}| - 1 \tag{3}$$

where $A^k_{\mathcal{P}}$ denotes the subset of arcs with $f_k(a) > 0$ in $\mathcal{P}$ and forbidden clique constraints associated with a clique Q

$$\sum_{k \in Q} \sum_{a \in A^k_Q} f_k(a) \leq \sum_{k \in Q} |A^k_Q| - 1 \tag{4}$$

where A^k_Q is the subset of arcs a corresponding to edges in G where two paths from Q meet.

The objective function value cap computed by ILP (2) enhanced by constraints (3) and (4) does not necessarily equal the load bound $\ell(G, \mathcal{D})$ anymore, but corresponds to the maximal edge load of a most-balanced routing, taking forbidden cliques and forbidden previous routings into account.

That way, it is possible to increase the lower bound towards a match with the current upper bound, coming from the spectrum width of the best solution found so far.

An edge-path formulation (EPF). For the framework to compute $\chi_I(G, \mathcal{D})$, any edge-path formulation to solve the RSA problem can be used. Here we make use of the edge-path formulation related to the novel edge-node model from [7], i.e., we will adopt the way to encode the spectrum assignment from [7], but will simplify the routing as follows.

Let $\bar{\mathcal{P}}$ be the set of currently-considered routing paths, partitioned into $\bar{\mathcal{P}} = \bar{\mathcal{P}}_1 \cup \ldots \cup \bar{\mathcal{P}}_{|\mathcal{D}|}$ where $\bar{\mathcal{P}}_k = \{P^1_k, \ldots, P^{m_k}_k\}$ denotes the subset of routing paths currently available for demand

$k \in \mathcal{D}$. To find a routing $\mathcal{P}$, we use path selection variables

$$y_k^i = \begin{cases} 1 & \text{if path } P_k^i \in \bar{\mathcal{P}}_k \text{ is selected for } \mathcal{P}, \\ 0 & \text{otherwise,} \end{cases}$$

and have to ensure that one path per demand is taken.

For the spectrum assignment, we adopt the following sets of binary variables from [7]: For each demand $k \in \mathcal{D}$ and each edge $e \in E$, variable $x_e^k \in \{0,1\}$ indicates whether or not demand k is routed through edge e. For each demand $k \in \mathcal{D}$ and each slot $s \in S$, variable $z_s^k \in \{0,1\}$ encodes the fact of whether or not the slot s is the last slot of the channel assigned to demand k. For each demand $k \in \mathcal{D}$, each slot $s \in S$ and each edge $e \in E$, variable $t_e^{sk} \in \{0,1\}$ indicates whether or not demand k uses the slot s on edge E. Moreover, a variable $s_{max} \in S$ is used to express the maximum slot used.

Let b_{low} be the current lower bound and b_{up} be the current upper bound on $\chi_I(G,\mathcal{D})$, then the edge-path formulation based on the edge-node model from [7] reads as minimum violation problem:

$$\begin{array}{ll} \min |\mathcal{D}| s_{max} & + \sum_{b_{low} < s < b_{up}, k \in \mathcal{D}} z_k^s \\ \sum_{P_k^i \in \bar{\mathcal{P}}_k} y_k^i & = 1 \ \forall k \\ \sum_{P_k^i \in \bar{\mathcal{P}}_k, e \in P_k^i} y_k^i & = x_k^e \ \forall k, e \\ \sum_{1 \le s < w_k} z_k^s & = 0 \ \forall k \\ \sum_{w_k \le s < b_{up}} z_k^s & = 1 \ \forall k \\ \sum_{1 \le j \le w_k} z_k^{s+j} + x_k^e & \le t_k^{e,s} + 1 \ \forall k, e, s \in \{1, \dots, b_{up}-1\} \\ \sum_{1 \le s < b_{up}} t_k^{e,s} & = w_k x_k^e \ \forall k, e \\ \sum_{k \in \mathcal{D}} t_k^{e,s} & \le 1 \forall e, s \in \{1, \dots, b_{up}-1\} \\ \sum_{w_k \le s < b_{up}} s z_k^s & \le s_{max} \ \forall k \\ s_{max} & \le b_{up} - 1 \\ y_k^i, x_k^e, z_k^s, t_k^{e,s} & \in \{0,1\} \end{array} \tag{5}$$

The objective function ensures that a span-minimal solution is found and that the use of frequency slots within $\{b_{low}+1, \dots, b_{up}-1\}$ is penalized. The path selection constraints ensure that one path per demand is selected, the remaining constraints are adopted from [7] where it was shown that they correctly encode a solution (when all demands have to be served).

Note that for the first run of the edge-path formulation, the intitial values are $b_{low} = \ell(G,\mathcal{D})$ and $b_{up} = \bar{s}+1$ so that we operate on the full spectrum $\{1, \dots, \bar{s}\}$; each subset $\bar{\mathcal{P}}_k$ contains exactly one path, obtained from the initial routing $\mathcal{P}$. If the first run of the edge-path formulation does not result in a solution, we have to go back to the multi-commodity flow to find another routing. If a solution $(\mathcal{P},\mathcal{S})$ with span s_{max} has been found, we proceed as follows: if s_{max} equals b_{low}, then $(\mathcal{P},\mathcal{S})$ is clearly optimal; otherwise, we have $b_{low} < s_{max} < b_{up}$, update $b_{up} = s_{max}$ and keep $(\mathcal{P},\mathcal{S})$ as currently best solution.

To analyze the solution $(\mathcal{P},\mathcal{S})$ in terms of cliques of weight greater than b_{low}, we proceed as follows. Determine from $(\mathcal{P},\mathcal{S})$ the subset $\mathcal{D}_c \subset \mathcal{D}$ of critical demands k whose channel $S_k \in \mathcal{S}$ uses frequency slots within $\{b_{low}+1, \dots, b_{up}-1\}$. Critical demands k may be contained in a clique Q of weight $w(Q) > b_{low}$, and this clique Q must be contained in the closed neighborhood $N[k] = N(k) \cup \{k\}$ of k in the edge intersection graph $I(\mathcal{P})$ of the routing $\mathcal{P}$. Hence, for each critical demand $k \in \mathcal{D}_c$, we construct the subgraph H_k of $I(\mathcal{P})$ induced by $N[k]$, enumerate in H_k all cliques Q of weight $w(Q) > b_{low}$ and include them in a set $\mathcal{Q}$ of forbidden cliques as triples $(\mathcal{P}_Q, E_Q, w(Q))$ with $\mathcal{P}_Q = \{P_k \in \mathcal{P} : k \in Q\}$ and E_Q subset of edges of G where paths from $\mathcal{P}_Q$ meet.

In later runs of the edge-path formulation, the use of the spectrum $\{1, \dots, b_{up}-1\}$ ensures that every new solution improves the current upper bound. In addition, we have to take forbidden cliques Q (of weight $w(Q) > b_{low}$) and forbidden routings (to never consider a same routing twice) into account. For that, (5) is enhanced by forbidden (partial or full) routing constraints: for a forbidden clique or a forbidden routing $\mathcal{P}'$,

$$\sum_{P_k^i \in \mathcal{P}'} y_k^i \le |\mathcal{P}'| - 1 \tag{6}$$

ensures that not all paths $P_k^i \in \mathcal{P}'$ can be selected together again. Note that if $\mathcal{P}'$ corresponds to a clique, then all routings containing this subset of paths are forbidden, to exclude all routings $\mathcal{P}$ with $\omega(I(\mathcal{P}), \mathbf{w}) > b_{low}$. If in later runs of the edge-path formulation, the current lower bound b_{low} is larger than the weight of a forbidden clique Q, then the clique Q has to be reallowed to operate on the whole set of routings with b_{low} as lower bound. For that, an intermediate value b_q indicating the weight of the lightest forbidden clique will be used.

Framework to compute $\chi_I(G,\mathcal{D})$. Here, we summarize the results from the previous sections to formulate a framework to compute $\chi_I(G,\mathcal{D})$.

Input: We take as input an instance $(G, \bar{s}, \mathcal{D})$.
Output: The output will be a solution $(\mathcal{P}^*, \mathcal{S}^*)$ with span $\chi_I(G,\mathcal{D})$ or a certificate for infeasibility.
Initialization: We initialize

- an upper bound by $b_{up} = \bar{s}+1$, a clique bound by $b_q = \bar{s}+1$,
- a set of previously used routings by $\mathcal{F} = \emptyset$, and a set of critical non-edge cliques by $\mathcal{Q} = \emptyset$.

We construct the auxiliary network G_f from G and compute in G_f a multi-commodity flow f using ILP (2) with the objective to minimize *cap*.

If no feasible solution has been found then

- return "instance infeasible (due to transmission reach)"

Else (flow f with capacity *cap* has been found):

- if $cap > \bar{s}$ return "instance infeasible (as $\ell(G,\mathcal{D}) > \bar{s}$)"
- else set lower bound $b_{low} = cap$
  determine from f the according routing $\mathcal{P}_f$
  initialize a set of considered paths by $\bar{\mathcal{P}} = \mathcal{P}_f$.

Edge-Path Formulation (EPF): Launch the edge-path formulation (5) with $\bar{\mathcal{P}}$ as set of paths, enhanced by forbidden (partial or full) routing constraints (6) for all $\mathcal{P}' \in \mathcal{Q} \cup \mathcal{F}$ as a minimum violation problem where the objective is to minimize the span of the solution and penalties.

If no feasible solution has been found:

- Let $\mathcal{F} := \mathcal{F} \cup \{\mathcal{P}_f\}$ and continue with MCF.

Else (i.e. a solution $(\mathcal{P},\mathcal{S})$ with span s_{max} has been found):

- If $s_{max} = b_{low}$, then return $(\mathcal{P},\mathcal{S})$ as optimal solution.

- Else update $b_{up} = s_{max}$ and keep $(\mathcal{P}, \mathcal{S})$ as currently best solution.
  Determine from $(\mathcal{P}, \mathcal{S})$ the subset $\mathcal{D}_c \subset \mathcal{D}$ of critical demands k whose channel $S_k \in \mathcal{S}$ uses frequency slots within $\{b_{low} + 1, \ldots, b_{up} - 1\}$. For each critical demand $k \in \mathcal{D}_c$:
  – Construct the subgraph H_k of $I(\mathcal{P})$ induced by $N[k]$, find in H_k all cliques Q of weight $w(Q) > b_{low}$.
  – Include them in $\mathcal{Q}$; if $w(Q) < b_q$ then update $b_q = w(Q)$.
  Let $\mathcal{F} = \mathcal{F} \cup \{\mathcal{P}_f, \mathcal{P}\}$ and continue with MCF.

Multi-Commodity Flow (MCF): Compute a multi-commodity flow f minimizing *cap* using ILP (2), enhanced by forbidden routing constraints (3) associated with $\mathcal{P} \in \mathcal{F}$ and forbidden clique constraints (4) associated with $Q \in \mathcal{Q}$. If no flow has been found:

- if $b_q = b_{up} = \bar{s} + 1$, return "instance infeasible ($\chi_I(G, \mathcal{D}) > \bar{s}$)"
- else if $b_{up} \leq b_q, \bar{s}$, return $(\mathcal{P}^*, \mathcal{S}^*)$ as optimal solution
- else (i.e. we have $b_{low} < b_q < b_{up} \leq \bar{s}$): remove from $\mathcal{Q}$ all cliques Q of weight $w(Q) = b_q$, update b_q to $\min(w(Q) : Q \in \mathcal{Q})$ or to $\bar{s} + 1$ if $\mathcal{Q} = \emptyset$, continue with MCF.

Else (i.e. a flow f with capacity *cap* has been found):

- if $b_q = b_{up} = \bar{s} + 1 \leq cap$, return "instance infeasible ($\chi_I(G, \mathcal{D}) > \bar{s}$)"
- else if ($b_q = b_{up} \leq \bar{s}$ and $b_{up} \leq cap$) or ($b_{up} < b_q$ and $b_{up} \leq cap$), return $(\mathcal{P}^*, \mathcal{S}^*)$ as optimal solution
- else (we make some updates and continue):
  if ($cap < b_q$ and $cap < b_{up}$), set $b_{low} = cap$,
  if ($b_q \leq cap$ and $b_q < b_{up}$), set $b_{low} = b_q$, remove from $\mathcal{Q}$ all cliques Q of weight $w(Q) = b_q$,
  update b_q to $\min\{w(Q) : Q \in \mathcal{Q}\}$ or to $\bar{s} + 1$ if $\mathcal{Q} = \emptyset$,
  determine from f the routing $\mathcal{P}_f$, add the paths from $\mathcal{P}_f$ to $\bar{\mathcal{P}}$ and continue with EPF.

With the help of the above given arguments and a case analysis of the possible situations after running the multi-commodity flow, we can show:

THEOREM 2.2. *Given an instance $(G, \bar{s}, \mathcal{D})$ of the RSA problem, the above described framework correctly computes a solution $(\mathcal{P}^*, \mathcal{S}^*)$ with span $\chi_I(G, \mathcal{D})$ or certifies infeasibility.*

3 COMPUTATIONAL RESULTS

In this section, we present some preliminary computational results to evaluate the computational performances achieved with the herein proposed framework for the use of edge-path formulations in comparison with the related edge-node formulation from [7] (i.e., where both formulations use the same way to encode the spectrum assignment).

For that, we use two different sets of test instances. On the one hand, we use artificially constructed test instances $(G, \bar{s}, \mathcal{D})$ with networks having up to 14 nodes and only few demands, some of them having the property that $\ell(G, \mathcal{D}) < \chi_I(G, \mathcal{D})$, some being infeasible due to $\bar{s} < \chi_I(G, \mathcal{D})$, indicated by "inf." in Table 1.

On the other hand, three network topologies from the literature are investigated: Spain, NSF and German [14, 17]. The Spain topology has 21 nodes, 35 edges and 50 slots; the NSF topology has 14 nodes, 21 edges and 60 slots; the German topology has 17 nodes, 25 edges and 60 slots. For each network topology, three sets of randomly generated demands are evaluated. Each considered demand requires either 3, 5 or 6 slots and supports, respectively, 100 Gb/s (3000 km reach), 200 Gb/s (1500 km reach), or 400 Gb/s (600 km reach). The computational results are listed in Table 2.

Network	$\bar{s}$	$\ell(G, \mathcal{D})$	$\chi_I(G, \mathcal{D})$	#	FWK (ms)	ENF (ms)
Test net 1	5	3	4	2	58	95
Test net 2	5	-	inf.	0	4	15
Test net 2	8	6	6	2	36	137
Test net 3	16	11	13	3	368	1182
Test net 4	20	11	16	9	1247	17738
Test net 5	16	12	14	4	454	1425
Test net 6	12	8	10	3	227	233
Test net 7	7	-	inf.	0	7	151
Test net 7	8	-	inf.	1	50	211
Test net 7	9	8	9	2	61	251
Test net 7	10	7	9	3	245	654

Table 1: Comparison between the two approaches on artificially constructed test instances.

Network	$\bar{s}$	$\vert\mathcal{D}\vert$	$\ell(G, \mathcal{D})$	$\chi_I(G, \mathcal{D})$	#	FWK (ms)	ENF
Spain	50	10	4	4	2	20523	n.t.
Spain	50	20	7	7	9	647694	n.t.
Spain	50	30	8			n.t.	n.t.
German	60	10	12	12	4	241565	n.t.
German	60	20	23	23	2	190286	n.t.
German	60	30	32			n.t.	n.t.
NSF	60	30	32	32	2	100851	n.t.
NSF	60	60	38	38	2	60650	n.t.
NSF	60	90	50	50	2	2135706	n.t.

Table 2: Comparison between the two approaches on networks from the literature.

All experiments are performed using the state-of-the-art MIP solver CPLEX 12.10 on the high performance platform available at LIMOS. For each instance, the lower bound $\ell(G, \mathcal{D})$ and the minimum spectrum width $\chi_I(G, \mathcal{D})$ are given, followed by the number # of iterations needed by the framework FWK. For both formulations, the herein proposed framework (FWK) and the related edge-node formulation (ENF), the total time in milliseconds required for the optimization is displayed. A closer analysis of the overall running time spent by our framework FKW shows moreover that the percentage of the time spent solving the subproblems with the edge-path formulation increases with the number of demands (up to 99 % for the last instance in Table 2).

A time limit of 100 hours was imposed in each run. The absence of results for some instances indicates that the computation could not terminate within this time limit[2], indicated by "n.t." in Table 2.

We clearly see that, for all artificially constructed test instances, the computation time of the herein proposed framework is certainly smaller than for the related edge-node formulation from [7]. Moreover, the herein proposed framework could solve substantially more instances to optimality within the time limit than the related edge-node formulation, see Table 2.

[2] Unfortunately, the high performance platform does not return any information (e.g. on the objective function value of the best solution found) when the process exceeds the time limit so that no information about the remaining gap can be provided.

4 CONCLUDING REMARKS

In this paper, we studied the routing and spectrum assignment problem. The majority of existing models for the problem uses edge-path formulations where variables are associated with all possible routing paths so that the number of variables grows exponentially with the size of the instance. Therefore, either precomputed subsets of all possible paths per demand are used (which cannot guarantee optimality of the solutions) or column-generation methods have to be applied (as the explicit models are far too big for computation). However, computational results show that the size of the instances that can be solved to optimality that way is rather limited, see e.g. [2, 7, 11, 13, 15].

Our contribution is to provide a framework for the use of edge-path formulations to minimize the spectrum width of a solution. For that, we select an appropriate subset of paths to operate on with the help of combinatorial properties in such a way that optimality of the solution can be guaranteed due to a match of a lower bound (derived from the edge load of the routings) and an upper bound (coming from the span of the best solution found so far).

First computational results suggest that the herein proposed framework for the use of edge-path formulations is competitive in comparison with the related edge-node formulation from [7] (i.e., where both formulations use the same way to encode the spectrum assignment).

Our future work includes comparing different edge-path formulations from the literature (or edge-path formulations derived from edge-node formulations) to see which one behaves best in the context of our framework.

Moreover, there are different directions to further improve the current framework. On the one hand, we observe that two demands $k, k' \in \mathcal{D}$ with the same origin-destination pair operate on the same set of routing paths $\bar{\mathcal{P}}_k = \bar{\mathcal{P}}_{k'}$. If, in addition, $w_k = w_{k'}$ holds, then both the routes and the channels assigned to k and k' can be exchanged while keeping the same physical solution in the network. With an increasing number of demands, this effect causes a large number of symmetric solutions so that applying symmetry breaking techniques seems to be advantageous.

On the other hand, the cliques $Q \in \mathcal{Q}$ are used to prevent that all routings containing them are explored during the process. We note that all forbidden clique contraints (4) for MCF and (6) for EPF are redundant if they are associated with cliques $Q \in \mathcal{Q}$ properly contained in another clique $Q' \in \mathcal{Q}$. This observation shall be used to reduce the number of redundant constraints in order to speed up the computation.

Finally, our future work includes proposing similar frameworks to handle the RSA problem w.r.t. other objectives like minimizing the number of edges in routing paths, the lengths of the routing paths, or the number of edges from the network used to route the demands.

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