An Improved Single-Commodity Flow Formulation for the Vehicle Routing Problem with a Heterogeneous Fleet

Devanand
SnT, University of Luxembourg
Kirchberg, Luxembourg
devanand.devanand@uni.lu

ABSTRACT

The heterogeneous vehicle routing problem (HVRP), a variant of the classical vehicle routing problem (VRP), involves optimizing route planning for vehicles with different load capacities, each designed for specific tasks or constraints. Improving the HVRP model not only enhances the solution quality and reduces solving time but could also be useful for related extended versions such as HVRP with time windows, pickup and delivery, multiple depots, stochastic elements, and industry-specific constraints. In this paper, we present a novel formulation for HVRP that uses a single-commodity flow approach based on a 2-index formulation. In contrast to the conventional single-commodity flow formulation, our approach requires a significantly smaller number of variables. We performed a computational experiment to show the efficiency of our model by solving some HVRP instances and found a significant advantage.

1 INTRODUCTION

A classical vehicle routing problem (VRP) involves customers with specified item demands to be fulfilled by a identical fleet of vehicles. Here, all vehicles originate and conclude their routes at a common point, a depot. The primary aim is to minimize the combined distance covered by all vehicles while meeting the customers' demands. VRP has been extensively studied due to its direct economic and environmental importance in logistic and supply chain operations. The transportation process constitutes 10% to 20% of the ultimate cost of goods. Also, international freight transport accounts for around one-third of the total CO2 emissions [21]. Due to this, research on VRPs has always been demanding and growing exponentially [5]. VRP was first presented by Dantzig and Ramser [8]. The initial stage of VRP works often focused on developing mathematical models and exact algorithms for homogeneous fleets, serving as a foundation for later extensions. We refer reader [7, 14, 18] for various exact and heuristics techniques under such VRPs.

The VRP with a heterogeneous vehicle fleet, called heterogeneous VRP (HVRP), is a popular VRP that allows organizations to deploy various vehicles with different capacities, each tailored to specific tasks or constraints.

HVRP is formally described as follows: Suppose $G = (V, E)$ is a complete graph where $V = \{0, 1, \dots, n\}$ is a set of nodes, and $E = \{(i, j) \mid i, j \in V,\ i\ != j\}$ is the set of all possible edges between nodes. Here, $0 \in V$ represents a central depot, where all the vehicles start and return by serving all the customers. $N = V \backslash \{0\}$ represents the index set of customer location. The depot has τ number of vehicles whose capacities are denoted by a set $T = \{t_1, t_2, \dots, t_\tau\}$, where t_i denotes the maximum capacity i^{th} vehicle can carry. In the case of VRP with a homogeneous fleet of vehicles, capacities will be the same, $t_i = t_j$; otherwise, at least one vehicle will be different by capacity. For each arc $(i, j) \in E$, we have a transportation cost $c_{i,j}$ to travel from customer location i to location j. For simplicity, we consider $c_{i,j}$ as the distance between location i and j. For each $i \in V$, we represent associated pickup quantities by $p_i \geq 0$. Our objective is to determine the vehicle routes from the depot 0 to customer points so that

- total cost be as minimum as possible
- total number of vehicles used as small as possible
- each customer is visited exactly once
- all the vehicles start from and end at the depot
- all customer demands must served by the vehicle

The pickup demand at each demand point is known before departure, and it can not be split. We assume that any vehicle can serve any of the customers. It means the restriction that a specific sized vehicle only serves a particular customer is not considered in our problem. The objective is to determine the optimal vehicle routes to pick up goods after reaching the demand point without violating the vehicle's maximum carrying capacities. We limit our focus to the basic HVRP with a fixed number of vehicles, each with varying capacity constraints.

HVRPs have received greater attention in the literature. It was first studied in the seminal work of Golden, *et al.* [13] and has since developed into an extensive field of research. Detailed surveys of HVRP are conducted by [16] and cover the 30 years of development since HVRP was developed by Golden, *et al.*. There are works by [1, 2] that cover the solution strategy of HVRP. The model studied in the rich-VRP literature can be found in [6].

The HVRP is NP-hard as it is a natural generalization of the travelling salesman problem (TSP). Many heuristics and exact methods have been proposed in the literature. Classical heuristics leverage extensions from well-established heuristics for classical VRPs [12, 22, 23]. Tabu search-based heuristics, extensively tested and studied, have proven effective for HVRPs [10, 25]. For branch-and-cut and branch-price-and-cut are the main approaches that depend on the modeling of the HVRP. We can find the work of lower bounding and its variants for HVRP in [2]. [20] studies exact solving procedures using branch-and-price-and-cut for VRP.

For exact branch-and-bound algorithms, [17, 19] have made significant contributions; however, these algorithms tend to work optimally only on relatively small instances. In contrast, Baldacci and Mingozzi [4] present an exact algorithm for the HVRP, which

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generalizes bounding procedures and exact methods described for the CVRP. They introduce novel bounding methods demonstrating particular effectiveness when the vehicle's fixed cost contribution to the total cost is significant. Various valid inequalities and delayed constraints specific to the problem are employed to speed up the solving procedure, such as capacity cuts, comb cuts, etc. [3, 26].

Researchers have proposed various mathematical models for the HVRP. Broadly, three variants of formulation for HVRPs are studied.

The first is based on single-commodity flow formulation [2]. In this, the entire vehicle fleet is considered as a single commodity. This formulation is based on the flow of the commodity from the depot to the customers and back to the depot. The objective is to minimize the total cost, often a combination of travel distances, vehicle fixed costs, and other relevant factors. The second type is the two-flow formulation of [3]. In this formulation, it is assumed that the vehicle types do not dominate and are ordered. The network of customers and depots is considered symmetric. In addition, a dummy depot is included in the modeling. Another type of formulation is the set partitioning-based formulation. In the set partitioning formulation of [4], given an undirected graph $G(V, E)$, each route is assumed to be a subset of the customer set V associated with cost, and the goal is to select the set of routes such that the union of all routes is V, subject to the associated VRP constraints. In this paper, we focus on the single-flow formulation.

Two-index vehicle flow models have found application in representing basic versions of classical symmetric and asymmetric-VRP (AVRP), including variants like the VRP with backhauls [24]. However, as the complexity of VRPs increases, these models may prove insufficient. To solve this problem, we explicitly specify the vehicle traveling through an arc so that more complex constraints can be imposed on the routes. This explicit representation allows the imposition of complicated constraints on the routes and provides a more flexible and robust solution for solving complex VRPs.

This paper contributes a novel formulation, a 2-index commodity flow model, which has fewer variables and constraints than the existing formulations of VRPSPDs, which are generally modeled as a 3-index commodity flow formulation. We discuss the existing model for VRPSPD in Section 2. Our new model is discussed in Section 3. In Section 4, we show computational results that highlight the advantages of our model. Finally, we summarize our work in Section 5 and give an outlook on future work.

2 EXISTING SINGLE-COMMODITY FLOW MODEL FOR HVRP

We follow the model of F. Gheysens *et al.*. [11]. It is a 3-index commodity flow formulation, the most widely used model, and is also suitable for modeling other complex classes of VRP.

The parameters and sets used in the model are the same as discussed above. For heterogeneity of vehicles, the capacity of some vehicles is non-identical. So, for some $t_i,\ t_j \in T, t_i \neq t_j$. Broadly, two categories of decision variables for the model are as follows:

- $x_{k,ij} = 1$ if vehicle k moves from customer i to j. Otherwise, it takes a 0 value.
- $pv_{i,j}$ is the decision variable that indicates the total load a vehicle carries while traversing from customer i to customer j for pickup of the items.

The problem is modeled as follows:

$$\text{3-HVRP } := \min \sum_{k=1}^{\tau} \sum_{i=0}^{n} \sum_{j=0}^{n} x_{k,ij}\ cost_{i,j}$$

$$\text{s.t.} \sum_{k=1}^{\tau} \sum_{i \in V, j \neq i} x_{k,ij} = 1\ \forall j \in N, \tag{1}$$

$$\sum_{j \in V} x_{k,ij} - \sum_{j \in V} x_{k,ji} = 0\ \forall i \in V,\ \forall k \in \{1 \ldots, \tau\}, \tag{2}$$

$$\sum_{i \in V} pv_{i,j} + p_j = \sum_{i \in V} pv_{j,i} \quad \forall j \in N, \tag{3}$$

$$\sum_{j \in N} x_{k,0j} \leq 1 \quad \forall k \in \{1, \ldots, \tau\}, \tag{4}$$

$$\sum_{i \in N} x_{k,i0} \leq 1 \quad \forall k \in \{1, \ldots, \tau\}, \tag{5}$$

$$pv_{i,j} \leq \sum_{k=1}^{\tau} t_k x_{k,ij}\ \ \forall i \in V,\ \forall j \in V, \tag{6}$$

$$x_{k,ij} \in \{0, 1\},\ \forall i \in V,\ \forall j \in V,\ \forall k \in \{1, \ldots, \tau\}, \tag{7}$$

$$pv_{i,j} \geq 0, \forall i \in V, \forall j \in V,$$

The model's objective function is to minimize the cost associated with the edges covered by vehicles. Constraint (1) ensures that each customer is served exactly once by one of the vehicles. Constraint (2) guarantees that the same vehicle enters and leaves the customer point. Flow conservation restrictions for pickup are mentioned in constraint (3). Constraints 4 and 5 represent flow-in and flow-out vehicles at the depot, which limits each vehicle to be used for a maximum of one route. Constraint (6) imposes the capacity constraint. It is a Miller-Tucker-Zemlin constraint [9] that handles the problem of subtour in the solution. Trivial constraints on binary and continuous variables are imposed in (7).

The model consists of variables having three indices. Clearly, such model requires $O(n^2 \cdot \tau)$ variables.

3 IMPROVED FORMULATION

3.1 Motivation

The existing model mentioned in the previous section explicitly tracks the vehicle type to serve a given customer. So, if $x_{k,ij} = 1$. It makes it very clear that the vehicle k served (reached) the customer (depot) j after serving (starting) from i.

Consider we have at least one feasible solution for a given HVRP problem. Without loss of generality, we can consider $0 - 1 - 2 - 3 \ldots, n - 0$ as a feasible path. The consequent solution obtained from the model 3-HVRP is $x_{1,01} = x_{1,12} = x_{1,23} = \ldots = x_{1,n0}$. It implies that vehicle 1 serves all the customers in the lexicographical order. For other vehicle, the variables $x_{t,n0}, \forall t \in \{2, \ldots, \tau\}$, take the value 0. If we replace $x_{k,ij}$ with a binary decision variable $x_{i,j}$, for the given feasible path, $x_{0,1} = x_{1,2} = x_{2,3} = \ldots = x_{n,0}$. However, we needed the information about the vehicle type which served all the customers. If $x_{n,0} = 1$ and vehicle k serves customer n associated to the feasible path, then it will serve all the remaining customers $j = \{1, \ldots, n-1\}$ to this path.

We can generalize this idea by introducing a new binary decision variable $y_{j,k}$ that represents whether the vehicle k serves the feasible paths such that the last visited customer is j. Clearly, $y_{j,k} = 1$ if $x_{j,0}$ is 1 and vehicle number k is used to carry the load associated at customer points $j \in J$. Where J is the set of all last visited customer point by any vehicles. That is, $J = \{j \in N \mid x_{j,0} = 1\}$. So, $\sum_{k=1}^{\tau} y_{j,k} = 1,\ j \in J$. Since we do not have any information about J before computing the model, we can rewrite it as -

$$\sum_{k=1}^{\tau} y_{j,k} = x_{j,0},\ \forall j \in N. \tag{8}$$

Equation 8 ensures that if the last visited customer is not j then $y_{j,k} = 1$ for any $k \in \{1, \ldots, \tau\}$. One important condition related to $y_{j,k}$ is the following:

$$\sum_{j \in N} y_{j,k} \leq 1,\ \forall k \in \{1, \ldots, \tau\}. \tag{9}$$

This ensures that a vehicle $k \in \{1, \ldots, \tau\}$ can not be connected to more than one feasible path.

Now we can associate the variables $y_{j,k}$ and $pv_{j,0}$ that form the maximum carrying capacity of vehicle k,

$$p_{var_{i,0}}\ y_{i,k} \leq t_k,\ \forall i \in N,\ \forall\ k \in \{1, \ldots, \tau\}. \tag{10}$$

Equation (10) consists nonlinear terms. We can linearlize the product of binary and continuous variables $z = p_{var_{i,0}}\ y_{i,k} \leq t_k$ as follows:

$$\begin{aligned} &z \leq y_{i,k} M, \\ &z \leq p_{var_{i,0}}, \\ &z \geq p_{var_{i,0}} + (1 - y_{i,k}) M, \\ &0 \leq z \leq t_k. \end{aligned} \tag{11}$$

Here M is a suitable large number. Since most of the optimization solvers also handle logical constraints, an alternative to the systems of Equations (11), we can have the following constraints:

$$y_{i,k} = 1 \implies p_{var_{i,0}} \leq t_k. \tag{12}$$

3.2 Model

Using the above constraints and other HVRP constraints, we form the 2-index formulation as follows:

$$\text{2-HVRP} := \min \sum_{i=0}^{n} \sum_{j=0}^{n} x_{i,j}\ cost_{i,j}$$

$$\text{s.t.} \sum_{i \in V, j \neq i} x_{i,j} = 1 \quad \forall j \in N, \tag{13}$$

$$\sum_{j \in V, i \neq j} x_{i,j} = 1 \quad \forall i \in N, \tag{14}$$

$$\sum_{i \in V} pv_{i,j} + p_j = \sum_{I \in V} pv_{j,i} \quad \forall j \in N, \tag{15}$$

$$\sum_{i \in N} pv_{0,i} = 0, \tag{16}$$

$$\sum_{i \in N} pv_{i,0} = \sum_{j \in N} p_j, \tag{17}$$

$$\sum_{k=1}^{\tau} y_{j,k} = x_{j,0},\ \forall j \in N \tag{18}$$

$$\sum_{j \in N} y_{j,k} \leq 1,\ \forall k \in \{1, \ldots, \tau\}, \tag{19}$$

$$y_{i,k} = 1 \implies p_{var_{i,0}} \leq t_k,\ \forall i \in N,\ \forall\ k \in \{1, \ldots, \tau\}, \tag{20}$$

$$\begin{aligned} &x_{i,j} \in \{0,1\},\ \forall i \in V,\ \forall j \in V, \\ &pv_{i,j} \geq 0, \forall i \in V, \forall j \in V, \\ &y_{i,k} \in \{0,1\},\ \forall i \in N, \forall k \in \{1, \ldots, \tau\}. \end{aligned} \tag{21}$$

Here, similar to the model 3-HVRP , the objective function in the model is to minimize the cost associated with the edges covered by vehicles. The indegree and outdegree constraints (13) and (14) ensure that exactly one entry and exit is allowed at each customer. Constraint (15) guarantees flow conservation restrictions for pickup. Equation (16) and (17) make sure that vehicles start empty from the depot and return with all the picked-up items to the depot. The details of constraints (18), (19) and (20) are provided in Section 3.1. Trivial constraints on binary and continuous variables are imposed in (21). Note that the model consists of variables having only two indices - such a model requires $O(n^2)$ variables and $O(n^2)$ constraints.

The model can be helpful to other classes of VRP problems with more added constraints - we can have a similar formulation for HVRP with pickup and load. Since time-window-based restrictions in HVRPs do not require any variable related to the vehicle's capacity, our model can adapt to such time-window-based problems.

4 COMPUTATIONAL EXPERIMENTS

In this section, we give empirical evidence of the effectiveness of our 2-index-based model (2-HVRP) in some HVRP instances and provide computational details of them. We compare the performance of our model with that of the 3-HVRP by solving it with the 22.1.1.0 version of CPLEX, one of the fastest optimization solvers. Both models are coded in Python (version 3.8), utilizing the CPLEX Python API. This Python package within CPLEX facilitates access to the Callable Library from the Python programming language.

The hardware used for the computation is a Mac OS M2 chip, an 8-core CPU supporting a 10-core GPU with a 3.49 GHz CPU. To avoid multiple processes sharing common resources, we run one job at a time with the default settings of CPLEX API.

We generate 18 test instances for our experiment, each with different customers and vehicles. The locations of the vehicles and customers are two-dimensional coordinate points (x, y) that are randomly generated, where x and y are from a uniform distribution such that $x \sim U[a_1, a_2]$ and $y \sim U[b_1, b_2]$.

It should be noted that there are HVRP instances that have already been well studied and tested [15]. Our experiments focused on smaller data sets. This was a deliberate choice, as our current work does not focus on refining solution strategies, but aims to motivate readers for the effectiveness of using an improved model. Consequently, we applied our model to the optimization solver avoiding delving into developing an exact method for its solution.

The vehicle's capacity is chosen randomly and is uniformly distributed between q_1 to q_2. The value of items to be picked up by vehicle at the customer location is also randomly generated, uniformly distributed between p_1 to p_2. Selection of the number of vehicles should ensure sufficient supply to serve all the customers. The simple approach to estimate this value is always to keep the number of vehicles more than the sum of total items to be picked divided by the average capacity of a vehicle.

Our data set is generated with the following suitable values: $a_1 = b_1 = 0$, $a_2 = 200$, $b_2 = 100$, $p_1 = 1$, $p_2 = 5$, $q_1 = 5$ and $q_2 = 10$. Out of 18 test instances, we show the detailed specification of the first 4 instances in Tables 1 and 2. In Table 1, for each instances, we list n, number of customers, τ, maximum number of vehicles available, p_i, $i = 1, \dots, n$, pickup items at each customer locations , and t_j, $j = 1, \dots, \tau$, the maximum carrying capacities of each vehicle. Note that the capacity of vehicles and items to be picked at the customer ends have the same units. Each n_i, $i = 1, \dots, n$ in Table 2 is the $x{-}y$ coordinate that represents customer locations. For our experiments, we consider $cost_{i,j} = \left\|n_i - n_j\right\|_2$, the Euclidean distance between customers i and j. The remaining test instances consist of the following number of customer n and the number of vehicles τ:

$$(n, \tau) = \begin{cases} (8, 4), & \text{if } I = I5,\ I6,\ I7,\ I8, \\ (10, 6), & \text{if } I = I9,\ I10,\ I11,\ I12, \\ (15, 8), & \text{if } I = I13,\ I14,\ I15,\ I16, \\ (40, 20), & \text{if } I = I17,\ I18. \end{cases}$$

All test instances (in CSV)and models (in .LP format) are available on https://github.com/devanandR/HVRP.git.

Table 3 compares the performance of our 2-HVRP to the existing 3-HVRP . For both the models, we report optimal objective value, deterministic solving (wall) time taken by Cplex, and the number of vehicles used by the obtained optimal route, denoted by 'objval', 'time', and 'v-used', respectively. The unit for the time used is in seconds. We set the time limit of 600 seconds. The last column, 'improvement', compares the solving times. The first comparison, '%', reports the solving time benefit of using our method in terms of percentages. The second performance, 'times', measures how often our models are faster than the existing method. Compared to a 3-index-based model, our approach takes almost negligible time for small-sized instances. For a better picture of the effectiveness of our method, we refer to Figure 1. We use a log scale to show the solving time to illustrate the comparison. The blue column represents our model and the red column represents the existing model. The average time CPLEX takes to solve all the first 16 instances modeled as 2-HVRP is 0.56 seconds, much less than that of 3-HVRP , which takes 58 seconds.

The last two problem instances, I17 and I18, are chosen to be a difficult problem. Both the models hit the maximum time limit for I17 and I18. Interestingly, our model for such instances found a feasible integer solution with less than a 10 % optimality gap. However, the existing model could not find a feasible solution for such instances.

Table 1: Vehicles Capacities & Customer Demands

I	n	τ	p_1	p_2	p_3	p_4	p_5	t_1	t_2	t_3
I_1	5	3	1	1	2	2	1	6	9	7
I_2	5	3	2	2	4	4	3	5	7	7
I_3	5	3	4	1	2	1	3	8	9	9
I_4	5	3	1	2	2	3	3	9	6	8

Table 2: Customer Locations (the x-y-coordinate) for the Instances in Table 1

I		depot	n1	n2	n3	n4	n5
I1	x	7.946	199.811	122.772	24.721	125.448	167.456
	y	76.594	45.897	97.952	99.052	3.214	13.750
I2	x	192.64	140.37	170.73	82.36	114.75	59.00
	y	93.47	33.29	35.20	19.75	90.67	0.16
I3	x	88.51	134.67	148.54	126.48	180.12	94.98
	y	60.07	71.64	51.23	74.93	82.48	78.19
I4	x	62.95	141.06	15.64	113.02	64.45	17.54
	y	76.35	82.10	79.78	49.71	85.23	35.42

Table 3: Computational Summary of the Performance of our Model, 2-HVRP Compared to 3-HVRP

I	3-indexModel			2-index			improvement	
	objval	time	v-used	objval	time	v-used	%	times
I1	446.51	0.18	1	446.51	0.016	1	91.11	11.3
I2	654.37	0.023	3	654.37	0.02	3	13.04	1.2
I3	239.91	0.05	2	239.91	0.017	2	66.00	2.9
I4	327.52	0.1	2	327.52	0.022	2	78.00	4.5
I5	632.91	0.19	3	632.91	0.04	3	78.95	4.8
I6	353.16	0.31	3	353.16	0.076	3	75.48	4.1
I7	489.84	0.13	3	489.84	0.036	3	72.31	3.6
I8	431.13	0.15	3	431.13	0.06	3	60.00	2.5
I9	693.81	0.62	4	693.81	0.33	4	46.77	1.9
I10	665.15	1.32	4	665.15	0.3	4	77.27	4.4
I11	692.5	0.26	3	692.5	0.04	3	84.62	6.5
I12	618.5	0.66	3	618.52	0.12	3	81.82	5.5
I13	987.065	392.43	5	987.065	3.72	5	99.05	105.5
I14	818.7	45.5	5	818.7	0.31	5	99.32	146.8
I15	1319.46	6.46	5	1319.46	0.47	5	92.72	13.7
I16	1168.76	493.16	5	1168.76	3.45	5	99.30	142.9
I17	unsolved	timeout		2659.48 (9.88%)	timeout	15	-	-
I18	unsolved	timeout		2988(8.7%)	timeout	14	-	-

5 CONCLUSION AND FUTURE WORK

In this paper, we have presented a 2-index-based single-commodity flow model for the heterogeneous vehicle routing problem (HVRP). Our model has been shown to be a compelling alternative to the existing 3-index-based single-commodity flow model, as it requires significantly fewer variables. This reduction in complexity contributes to a computationally more efficient technique for solving the problem. When solving the models with CPLEX, an optimization solver, we saw remarkable speedups for smaller problem instances with up to 15 customers. In addition, for two larger instances with 40 customers, our model also showed impressive result by providing feasible integer solutions with a gap of less than 10% for instances where the existing model struggled to reach even a single integer feasible solution. Notably, our model can be extended to other classes of problems, including HVRP with time window constraints, pickup and delivery considerations, and other related HVRP extensions.

We have only tried to model the problem and focus on the basic version of the HVRP. Other complex constraints, such as HVRP with multiple depots, HVRP where some customers are only allowed to use certain types of vehicles, and related complex HVRP, are something that we are working towards.

The current work focuses only on modeling the problem. Our immediate research direction is to perform extensive computational experiments by exploiting valid inequalities and delayed constraints to solve large instances.

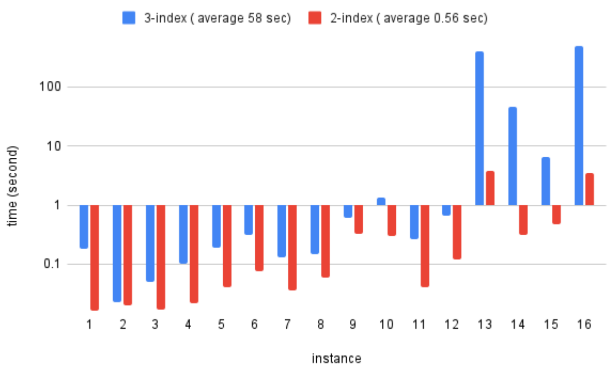

Figure 1: A Column Chart Comparing the Solving Time

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